UNIVERSITY *of* LIMERICK
OLLSCOIL LUIMNIGH

COLLEGE OF INFORMATICS AND ELECTRONICS

DEPARTMENT OF MATHEMATICS & STATISTICS

END OF SEMESTER ASSESSMENT PAPER

MODULE CODE: MS4414 **SEMESTER: Spring 2006-07**

MODULE TITLE: Theoretical Mechanics **DURATION OF EXAM: 2 hours**

LECTURER: Prof Eugene Benilov **GRADING SCHEME: Examination: 80%**

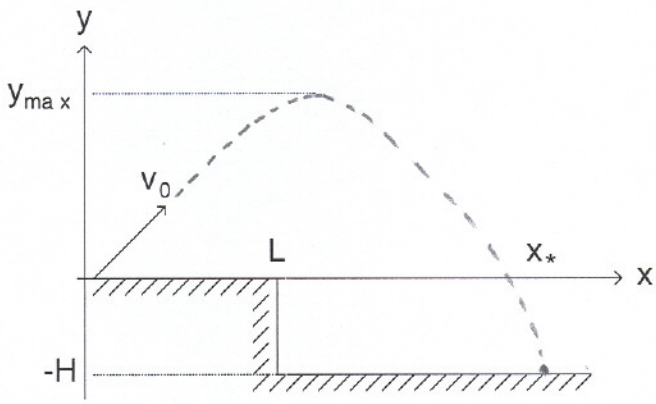
EXTERNAL EXAMINER: Prof J King

INSTRUCTIONS TO CANDIDATES

Please attempt all questions.

Question 1 [25 marks]

1) A stone is projected (under gravity, with air friction neglected) with velocity v_0, at an angle α to the horizon, towards a "pit" of depth H located at a distance L:



1.1) Determine for which v_0 the stone would reach the pit.

1.2) Assuming that the stone reaches the pit, calculate x_* (the x-coordinate of the point where the stone hits the ground) and y_{max} (the maximum height of its trajectory).

2) The trajectory of a particle is given by

$$x = \sin t, \qquad y = 2 \cos t, \qquad t: 0 \to \frac{3}{2}\pi.$$

Sketch the trajectory of the particle.

3) The same as in part 2, but for a particle with polar coordinates, given by

$$r = 2\pi - t, \qquad \theta = t, \qquad t: 0 \to 2\pi.$$

Question 2 [25 marks]

1) Three particles, of masses 4 kg, 3 kg, and 2 kg, simultaneously collide:

Before the collision, the middle particle was motionless, whereas the velocities of the other two were 1 m/s and -3 m/s (see the diagram). Assuming that the particles collide non-elastically and coalesce, find their velocity after the collision and the amount of mechanical energy lost in the collision.

2) A body of mass m is attached to the ceiling by a spring of unperturbed length L and modulus k.

2.1) Find the distance H_{eq} between the body's equilibrium position and the ceiling.

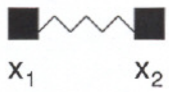
2.2) Assuming that the drag (friction) force is $F_{dr} = -\mu v$ (μ is the friction coefficient, v is the velocity of the body), derive an equation for the distance x(t) between the body's current and equilibrium positions.

2.3) Assuming that $m = 2$, $k = 8$, and $\mu = 4$, determine whether the oscillations of the body are underdamped, overdamped, or critically damped. Draw a typical graph of x(t).

3) Consider two planets of masses $m_{1,2}$ separated by a distance L, and an object located on the straight line connecting them. Where should the object be placed so that the gravity forces exerted on it by the planets are in exact balance?

Question 3 [25 marks]

1) Consider a one-dimensional system which consists of two particles of masses m_1 and m_2, with coordinates x_1 and x_2 ($x_1 < x_2$) connected by a spring of module μ and free length L:



Write down the expression for the Lagrangian of the system, and derive the Lagrangian form of the governing equations.

2) Consider the two-dimensional equivalent of the system described in Question 1 above, i.e. two particles of masses m_1 and m_2, with coordinates (x_1,y_1) and (x_2,y_2), connected by a spring of modulus μ and free length L.

Write down the expression for the kinetic and potential energies of the system and, thus, find its Hamiltonian.

3) Find and examine the fixed points of

$$\dot{\phi} = \phi + \psi^2 + \psi, \qquad \dot{\psi} = \phi + \psi^2 + 1.$$

Question 4 [25 marks]

Consider

$$\ddot{\phi} + 2c\,\dot{\phi} + (1 + \varepsilon \cos 2\Omega t)\,\phi = 0, \qquad (1)$$

where ε, $c \ll 1$ and $\Omega \approx 1$.

1) Seek a solution in the form

$$\phi = B(t) \cos \Omega t + D(t) \sin \Omega t. \qquad (2)$$

2) Upon substitution of (2) into (1), omit small terms involving $\ddot{B}$, $\ddot{D}$, $c\dot{B}$, and $c\dot{D}$.

3) Omit the non-resonant terms, i.e. terms involving $\cos 3\Omega t$ and $\sin 3\Omega t$.

4) Collect like terms and solve the resulting set of equations for $B(t)$ and $D(t)$.

5) Using these equations, determine the range of Ω for which parametric resonance occurs in the system.