

MS4414 Theoretical Mechanics

Tutorial week 12: Dynamical systems

Thursday 14 April 2011

Stability points

- ▶ System of two equations

$$\dot{\psi} = f(\phi, \psi) , \quad \dot{\phi} = g(\phi, \psi)$$

- ▶ Fixed points

$$0 = f(\phi, \psi) , \quad 0 = g(\phi, \psi)$$

- ▶ Stability: calculate the Jacobian matrix

$$\begin{bmatrix} \frac{\partial f}{\partial \phi} & \frac{\partial f}{\partial \psi} \\ \frac{\partial g}{\partial \phi} & \frac{\partial g}{\partial \psi} \end{bmatrix}$$

- ▶ Calculate the eigen-values. study the real parts
 - ▶ all negative: stable
 - ▶ one positive: unstable
 - ▶ at least one 0, other values negative: inconclusive

Question 1

► Equations

$$\dot{\phi} = \psi, \quad \dot{\psi} = \phi^2 + \phi\psi - 1$$

► Matrix

$$\begin{bmatrix} 0 & 1 \\ 2\phi + \psi & \phi \end{bmatrix}$$

► Fixed points

$$\psi = 0, \phi = 1 \quad \lambda_1 = -1, \lambda_2 = 2 \quad \textit{unstable}$$

$$\psi = 0, \phi = -1 \quad \lambda = \frac{-1 \pm i\sqrt{7}}{2} \quad \textit{stable}$$

Question 2

► Equations

$$\dot{\phi} = -\psi, \quad \dot{\psi} = \phi^2 - \phi\psi - 1$$

► Matrix

$$\begin{bmatrix} 0 & -1 \\ 2\phi - \psi & -\phi \end{bmatrix}$$

► Fixed points

$$\psi = 0, \phi = 1 \quad \lambda = \frac{-1 \pm i\sqrt{7}}{2} \quad \text{stable}$$

$$\psi = 0, \phi = -1 \quad \lambda_1 = -1, \lambda_2 = 2 \quad \text{unstable}$$

Question 3

► Equations

$$\dot{\phi} = \phi + \psi^2 + \psi, \quad \dot{\psi} = \phi + \psi^2 + 1$$

► Matrix

$$\begin{bmatrix} 1 & 1 + 2\psi \\ 1 & 2\psi \end{bmatrix}$$

► Fixed point

$$\psi = 1, \phi = -2 \quad \lambda = \frac{3 \pm \sqrt{13}}{2} \quad \textit{unstable}$$

Question 4

► Equations

$$\dot{\phi} = (\phi + 1)\psi , \quad \dot{\psi} = \phi(\psi - 1)$$

► Matrix

$$\begin{bmatrix} \psi & 1 + \phi \\ \psi - 1 & \phi \end{bmatrix}$$

► Fixed points

$$\psi = 0 , \phi = 0 \quad \lambda = \pm i \quad \textit{inconclusive}$$

$$\psi = 1 , \phi = -1 \quad \lambda_1 = -1 , \lambda_2 = -1 \quad \textit{unstable}$$