MS4414 Theoretical Mechanics
Tutorial week 5: Inelastic collisions

Thursday 9 February 2011

Inelastic collisions

- Conservation of momentum

$$\Sigma \mathbf{p}^{before\ collision} = \Sigma \mathbf{p}^{after\ collision}$$

- Momentum

$$\mathbf{p} = m\mathbf{v}$$

- Technique
  - Calculate the momentum for each particle before the collision
  - Calculate the mass of aggregated particles after the collision
  - Calculate the velocity using the conservation of momentum

Inelastic Collisions: question 1

- Momentum for the three particles

$$\begin{aligned} \mathbf{p_1} &= m_1\mathbf{v_1} = 1 \times 1\mathbf{i} = \mathbf{i} \\ \mathbf{p_2} &= m_2\mathbf{v_2} = 3 \times 0\mathbf{i} = \mathbf{0} \\ \mathbf{p_3} &= m_3\mathbf{v_3} = 2 \times (-3)\mathbf{i} = -6\mathbf{i} \end{aligned}$$

- Mass of the aggregated particles

$$m = m_1 + m_2 + m_3 = 1 + 3 + 2 = 6\ kg$$

- Conservation of momentum

$$\begin{aligned} & \mathbf{p} = \mathbf{p_1} + \mathbf{p_2} + \mathbf{p_3} \\ \iff & 6\mathbf{v} = \mathbf{i} + \mathbf{0} - 6\mathbf{i} \\ \iff & \mathbf{v} = -\frac{5}{6}\mathbf{i} \end{aligned}$$

Inelastic Collisions: question 2

- Momentum for the three particles

$$\begin{aligned} \mathbf{p_1} &= m_1\mathbf{v_1} = 4 \times 1\mathbf{i} = 4\mathbf{i} \\ \mathbf{p_2} &= m_2\mathbf{v_2} = 3 \times 0\mathbf{i} = \mathbf{0} \\ \mathbf{p_3} &= m_3\mathbf{v_3} = 2 \times (-3)\mathbf{i} = -6\mathbf{i} \end{aligned}$$

- Mass of the aggregated particles

$$m = m_1 + m_2 + m_3 = 4 + 3 + 2 = 9\ kg$$

- Conservation of momentum

$$\begin{aligned} & \mathbf{p} = \mathbf{p_1} + \mathbf{p_2} + \mathbf{p_3} \\ \iff & 9\mathbf{v} = 4\mathbf{i} + \mathbf{0} - 6\mathbf{i} \\ \iff & \mathbf{v} = -\frac{2}{9}\mathbf{i} \end{aligned}$$

Inelastic Collisions: question 3

- Momentum for the two particles

$$\begin{aligned} \mathbf{p_1} &= m_1 \dot{\mathbf{r}}_\mathbf{1} \\ \mathbf{p_2} &= m_2 \dot{\mathbf{r}}_\mathbf{2} \end{aligned}$$

- Momentum

$$\mathbf{p} = m_1 \dot{\mathbf{r}}_\mathbf{1} + m_2 \dot{\mathbf{r}}_\mathbf{2}$$

- Time dependence

$$\begin{aligned} \frac{d\,\mathbf{p}}{dt} &= m_1 \ddot{\mathbf{r}}_\mathbf{1} + m_2 \ddot{\mathbf{r}}_\mathbf{2} \\ &= \mathbf{F_1} + \mathbf{F_2} \\ &= \mathbf{0} \end{aligned}$$

Inelastic Collisions: question 4(1)

- Momentum for the two particles

$$\begin{aligned} \mathbf{p_1} &= m_1\mathbf{v_1} \\ \mathbf{p_2} &= m_2\mathbf{v_2} = 2m_1\mathbf{v_2} \end{aligned}$$

- Mass of the aggregated particles

$$m = m_1 + m_2 = m_1 + 2m_1 = 3m_1$$

- Conservation of momentum

$$\begin{aligned} & \mathbf{p} = \mathbf{p_1} + \mathbf{p_2} \\ \iff & 3m_1\mathbf{v} = m_1\mathbf{v_1} + 2m_1\mathbf{v_2} \\ \iff & \mathbf{v} = \frac{\mathbf{v_1} + 2\mathbf{v_2}}{3} \end{aligned}$$

Inelastic Collisions: question 4(2)

- Angle

$$\begin{aligned}\mathbf{v}\cdot\mathbf{v_1} &= |\mathbf{v}|\,|\mathbf{v_1}|\cos\alpha \\ &= \frac{\mathbf{v_1}+2\mathbf{v_2}}{3}\cdot\mathbf{v_1} = \frac{|\mathbf{v_1}|^2}{3}\end{aligned}$$

- Modulus

$$|\mathbf{v}| = \frac{\sqrt{|\mathbf{v_1}|^2 + 4\,|\mathbf{v_2}|^2}}{3}$$

- Cosine

$$\cos\alpha = \frac{\mathbf{v}\cdot\mathbf{v_1}}{|\mathbf{v}|\,|\mathbf{v_1}|} = \frac{|\mathbf{v_1}|}{\sqrt{|\mathbf{v_1}|^2 + 4\,|\mathbf{v_2}|^2}}$$