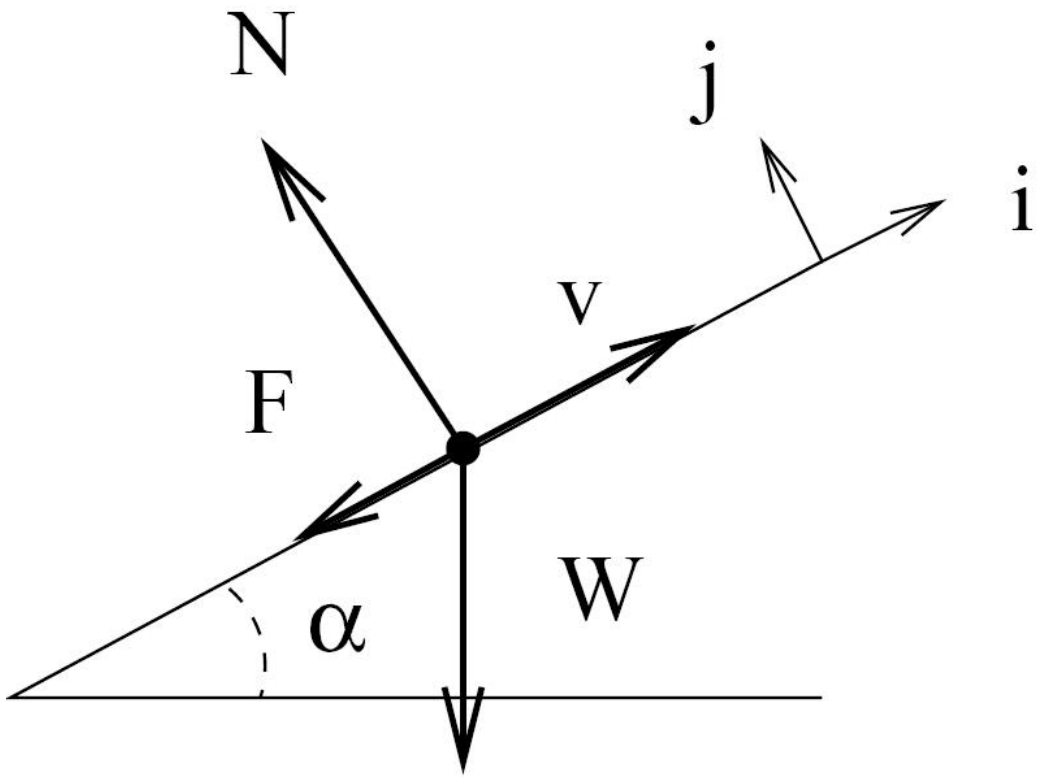
Friction: typical configuration



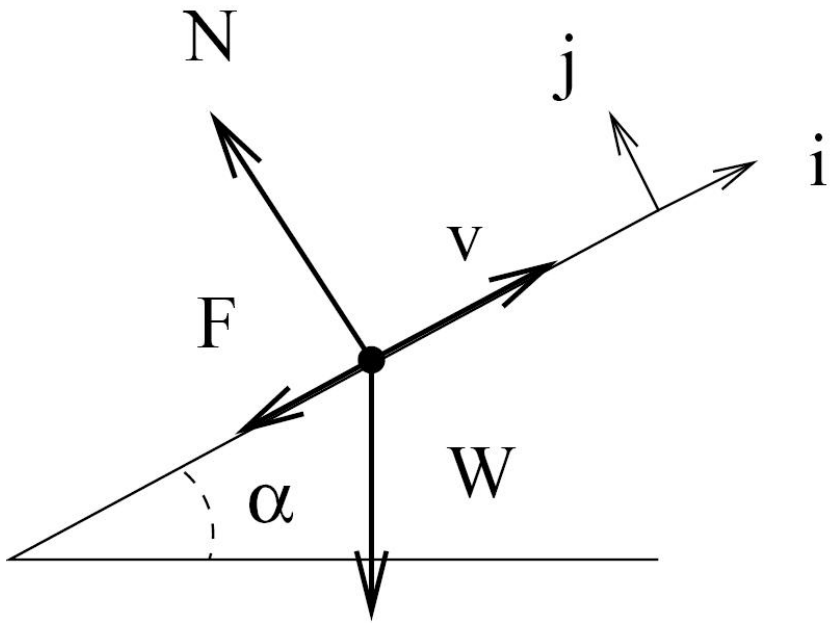
- Three forces
  - Weight W
  - Normal reaction R
  - Friction F
- Do a nice picture

Method

- Do a NICE graphic
- List all forces
- Apply Newton's second law
- Project on the movement direction
- Project on the direction perpendicular to movement
- Use the relation

$$F = fN\ , \qquad F > 0\ , \qquad N > 0$$

Question 1: Newton's second law



- Newton's second law

$$m\mathbf{a} = \mathbf{W} + \mathbf{N} + \mathbf{F}$$

- Projection direction $\mathbf{i}$ and $\mathbf{j}$

Question 1: projection and forces

- Project in the $\mathbf{i}$ direction

$$ma = -F - W\sin\alpha$$

- Project in the $\mathbf{j}$ direction

$$0 = N - W\cos\alpha$$

- Express the forces

$$\mathbf{W} = m\mathbf{g}\ , \qquad F = fN$$

- Express the acceleration

$$a = -g\left(\sin\alpha + f\cos\alpha\right)$$

Question 1: integration

- Velocity

$$v = -g\left(\sin\alpha + f\cos\alpha\right)t + v_0$$

- Position

$$x = -g\left(\sin\alpha + f\cos\alpha\right)\frac{t^2}{2} + v_0 t$$

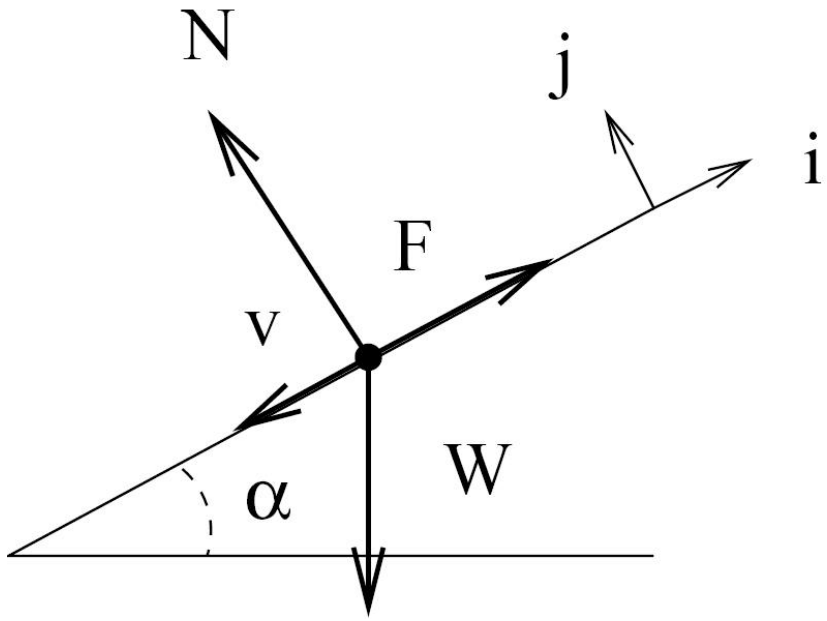
- Maximum height

$$v = 0 \quad \Longrightarrow \quad t = \frac{v_0}{g\left(\sin\alpha + f\cos\alpha\right)}$$

- Maximum height

$$x_{max} = \frac{v_0^2}{2g\left(\sin\alpha + f\cos\alpha\right)}$$

Question 2: Newton's second law



- Newton's second law

$$m\mathbf{a} = \mathbf{W} + \mathbf{N} + \mathbf{F}$$

- Projection direction $\mathbf{i}$ and $\mathbf{j}$

Question 2: projection and forces

- Project in the $\mathbf{i}$ direction

$$ma = F - W \sin \alpha$$

- Project in the $\mathbf{j}$ direction

$$0 = N - W \cos \alpha$$

- Express the forces

$$\mathbf{W} = m\mathbf{g} \ , \qquad F = fN$$

- Express the acceleration

$$a = g\left(f \cos \alpha - \sin \alpha\right)$$

Question 2: integration

- Conditions

$$v(0) = -v_0 \ , \qquad x(0) = 0$$

- Velocity

$$v = g\left(f\cos\alpha - \sin\alpha\right)t - v_0$$

- Position

$$x = g\left(f\cos\alpha - \sin\alpha\right)\frac{t^2}{2} - v_0 t$$

- Starting time

$$v = 0 \quad \Longrightarrow \quad t = \frac{v_0}{g\left(f\cos\alpha - \sin\alpha\right)}$$

- Starting position

$$x_{max} = -\frac{v_0^2}{2g\left(f\cos\alpha - \sin\alpha\right)}$$