Angular Momentum

Question 1 2005 Paper. The following system of four interconnected particles, of masses m and m' as shown on the diagram,



is rotating around its centre. Then, the distance L is reduced by a factor of 2. Using conservation of angular momentum, determine how L' needs to be changed to ensure that the angular velocity of the system remains the same as before.

Question 2 2006 Paper. Consider a system of 3 particles of masses m_1, m_2 , and m_3 , with position vectors \mathbf{r}_1 , \mathbf{r}_2 , and \mathbf{r}_3 , interacting with forces $\mathbf{F}_{1,2}$, $\mathbf{F}_{2,1}$, ..., $\mathbf{F}_{3,2}$. Prove that the angular momentum of the system with respect to the origin is conserved.

Question 3 2009 Paper. Consider a closed system of two particle of masses m_1 and m_2 located at \mathbf{r}_1 and \mathbf{r}_2 respectively. We suppose that the particles are moving (i.e. \mathbf{r}_1 and \mathbf{r}_2 depend on time t).

- (a) Write the momentum **P** of the system in terms of m_1 , m_2 , $\dot{\mathbf{r}}_1$, and $\dot{\mathbf{r}}_2$.
- (b) Show that \mathbf{P} does not depend on time.
- (c) Write the angular momentum \mathbf{A}_M of the system with respect to a fixed point M located at \mathbf{r}_M of the system in terms of $m_1, m_2, \mathbf{r}_M, \mathbf{r}_1, \mathbf{r}_2, \dot{\mathbf{r}}_1$, and $\dot{\mathbf{r}}_2$.
- (d) Supposing that the inter-particle interaction are parallel to the straight line going though the particles, show A_M does not depend on time.