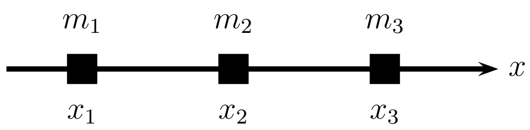
Energy and Gravitation

Question 1 Consider a one-dimensional system which consists of three particles of masses m_1, m_2, and m_3, with coordinates x_1, x_2, and x_3 ($x_1 \leq x_2 \leq x_3$), interacting with one another through gravity



(a) Using the one-dimensional version of Newton's Law of Gravity, determine the forces F_{12}, F_{21}, F_{13}, F_{31}, F_{23}, F_{32}, where F_{ij} is the force exerted on the j-th particle by the i-th particle.

(b) Using the relationship between the potential energy U of the system and the corresponding forces, show that the above expressions correspond to

$$U(x_1, x_2, x_3) = -\frac{Gm_1m_2}{x_2 - x_1} - \frac{Gm_2m_3}{x_3 - x_2} - \frac{Gm_3m_1}{x_3 - x_1}.$$

2005 paper.

Question 2 Consider two planets of masses $m_{1,2}$ separated by a distance L, and an object located on the straight line connecting them. Where should the object be placed so that the gravity forces exerted on it by the planets are in exact balance? *2007 paper.*

Question 3 A thin stick of mass m and length L is rotating around its end with angular velocity ω. Calculate the stick's kinetic energy. *2008 paper.*

Question 4 Two spherical objects, of radii $R_{1,2}$ and masses $m_{1,2}$, are attracted to each other through gravity. The initial velocities of the objects are zero, the initial distance separating them is infinitely large. Find their velocities when they collide. *2009 paper.*

Question 5 Two particles P_1 and P_2 having same mass m collide. We suppose that before collision particle P_1 was at rest while particle P_2 was moving with velocity $\mathbf{V}$ and that after collision, particle P_1 has velocity $\mathbf{v}_1$ and particle P_2 has a velocity $\mathbf{v}_2$. We suppose the collision is elastic.

(a) Prove that $v_1^2 + v_2^2 = V^2$

(b) Prove that the velocities of the two particles after collision, $\mathbf{v}_1$ and $\mathbf{v}_2$ are orthogonal.

2010 paper.

Question 6 We consider the Sun and Mars as two particles separated by a fixed distance d. The Sun is supposed fixed while Mars is rotating about it with a fixed angular velocity ω. We neglect all forces exerted on Mars except the gravitational attraction of the Sun. The mass of the Sun is M and the mass of Mars is m. We will consider a reference frame attached to the Sun.

(a) Write the relation between ω and the period of rotation of Mars around the Sun, T.

(b) Write the coordinates of Mars $(x(t)\,, y(t))$ in terms of ω, d, and t, supposing that $(x(0)\,, y(0)) = (d, 0)$.

(c) Find the coordinates (a_x, a_y) of the acceleration $\mathbf{a}$ of Mars in terms of ω, d, and t.

(d) Find the coordinates (F_x, F_y) of the gravitational force $\mathbf{F}$ exerted by the Sun on Mars in terms of M, m, the gravitational constant $\mathcal{G}$, d, ω, and t.

(e) Apply the second Newton law and find relation between ω, d, M, and $\mathcal{G}$.

(f) Having the following approximations, give an estimation of T expressed in days, $\mathcal{G} \approx 6.67 \times 10^{-11}\,\mathrm{m^3\,kg^{-1}\,s^{-2}}$, $d \approx 227 \times 10^9\,\mathrm{m}$, $M \approx 2 \times 10^{30}\,\mathrm{kg}$.

2010 paper.

Question 7 A meteorite, assimilated to a particle, starts moving towards the Earth from an infinitely large distance with an initial velocity v_0. We suppose that the mass m of the meteorite is very small compared to the mass M of the Earth and the meteorite is only subject to the gravitational attraction of the Earth. Express the velocity v_1 of the meteorite when it hits the surface of the Earth in terms of v_0, M, the radius of the Earth, R, and the gravitational constant, $\mathcal{G}$. *2010 paper.*