Trajectories

Question 1 *(2005 paper.)* The Cartesian coordinates of a particle on the plane are given by

$$x = t\cos t, \qquad y = -t\sin t, \qquad t : 0 \to \pi.$$

Find the velocity and acceleration of the particle, and sketch its trajectory (show as much detail as possible).

Question 2 *(2005 paper.)* Sketch the trajectory of a particle with polar coordinates, given by

$$r = 1 + t, \qquad \theta = -2t, \qquad t : 0 \to \pi.$$

Question 3 *(2006 paper.)* The trajectory of a particle is given by $x = 3\cos t$, $y = \sin t$ for $0 < t < \frac{3}{4}\pi$. Sketch the trajectory of the particle. Another particle has polar coordinates $r = 2\pi - t$, $\theta = t$ for $0 < t < 2\pi$. Sketch the trajectory of this particle.

Question 4 *(2007 paper.)*

(a) The trajectory of a particle is given by

$$x = \sin t, \qquad y = 2\cos t, \qquad t : 0 \to \tfrac{3}{2}\pi.$$

Sketch the trajectory of the particle.

(b) The same as in part (a) but for a particle with polar coordinates given by

$$r = 2\pi - t, \qquad \theta = t, \qquad t : 0 \to 2\pi.$$

Question 5 *(2008 paper.)* The trajectory of a particle is given by $x = t\cos t$, $y = \sin t$ for $0 < t < 3\pi$. Sketch the trajectory of the particle. Another particle has polar coordinates $r = \frac{1}{\cos t}$, $\theta = t$ for $0 < t < \frac{\pi}{2}$. Sketch the trajectory of this particle.