Introduction

MS4414 Theoretical Mechanics

William Lee

1 Module Description

Module MS4414, Theoretical Mechanics

Hours per week Lectures 2. Tutorial 1. Private study 7. ECTS Credits 6.

Grading type N.

Prerequisite Modules MS4613, MS4403.

Rationale and Purpose of the Module To introduce students to the fundamental concepts of theoretical mechanics. To prepare students by developing the basic mathematical skills in theoretical mechanics. To emphasise applications of vector calculus and ODEs.

Syllabus Kinematics: reference frames, motion in one dimension, motion with constant acceleration, kinematics in three dimensions, uniform circular motion, centripetal acceleration. Dynamics: mass, force, Newton's laws of motion, friction, Newton's law of gravity, planetary motion. Conservation laws: momentum, angular momentum, energy (kinetic energy, potential energy as gradient of force). Oscillatory motion: free and forced pendulum, resonance, parametric resonance. Introduction to the Hamiltonian and Lagrangian mechanics.

Learning Outcomes

1. Understand basic mechanical concepts, such as reference system, velocity, acceleration and force.
2. Understand and be able to apply Newton's Laws.
3. Understand and be able to apply the concepts of momentum and energy and their conservation.
4. Be able to apply Newton's Law of gravity to basic problems of celestial mechanics.
5. Understand the basic concepts of the Hamiltonian approach to mechanics, such as Hamiltonians, Hamiltonian equations, and Poisson brackets.
6. Understand the concept of stability and be able to apply it to basic mechanical problems.

Prime Texts

- P. Smith, R. C. Smith (1996). *Mechanics*, Wiley.
- F. Scheck (1999). *From Newton's Laws to Deterministic Chaos*, Springer.

2 Timetable

Monday	1700	Lecture	CO078	Weeks 1-9 and 11-14
Thursday	0900	Lecture	CO078	Weeks 1-9 and 11-14
Thursday	1300	Tutorial	S117	Weeks 1-9 and 11-14

Office Hours ???? A2016a.

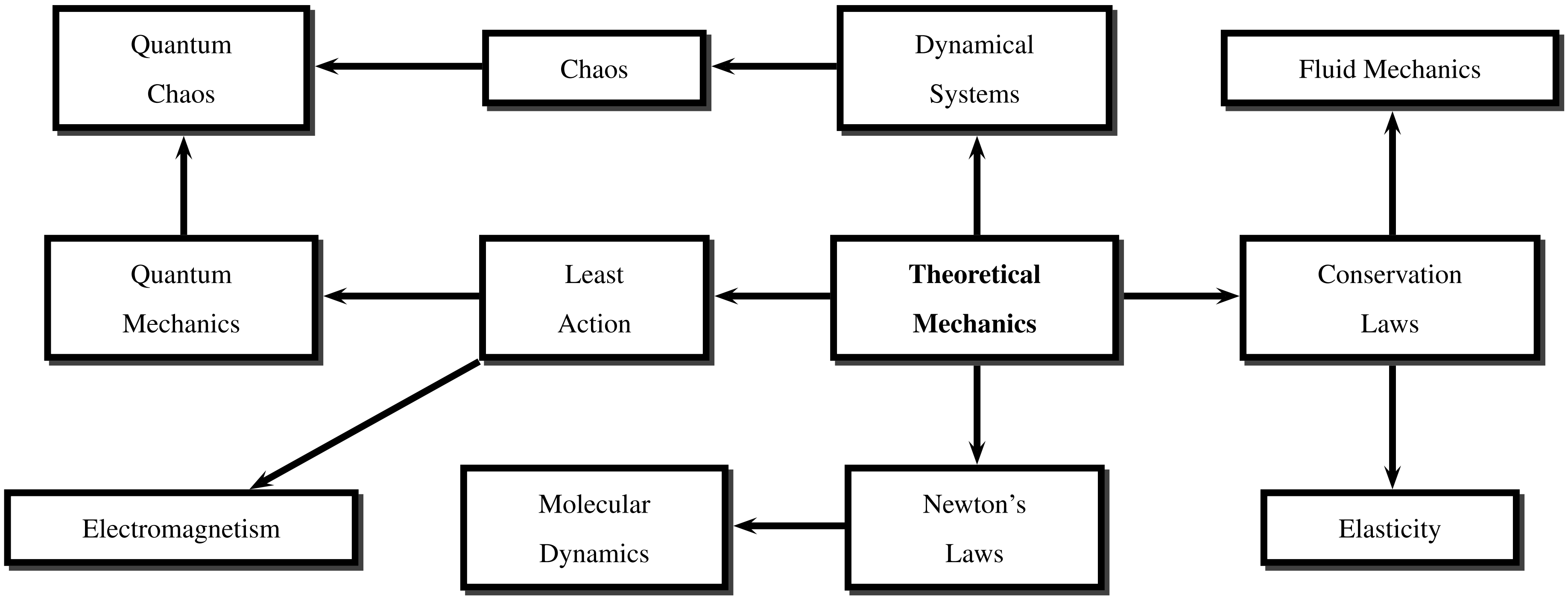
Advanced class ????.

3 Assessment

The course will be graded as follows:

20% of the marks will be given for a midterm paper.

80% of the marks will be given for a end of term exam.


Quantum Chaos
Chaos
Dynamical Systems
Fluid Mechanics
Quantum Mechanics
Least Action
Theoretical Mechanics
Conservation Laws
Electromagnetism
Molecular Dynamics
Newton's Laws
Elasticity

Theoretical Mechanics: Summary

Kinematics

1. Position vector $\mathbf{r}$, velocity $\mathbf{v}$, and acceleration $\mathbf{a}$ of a particle are related by:

$$\mathbf{v} = \dot{\mathbf{r}}, \qquad \mathbf{a} = \dot{\mathbf{v}} = \ddot{\mathbf{r}}.$$

2. 1D motion with constant velocity v:

$$x = vt + x_0.$$

3. 1D motion with constant acceleration a:

$$v = at + v_0, \qquad x = \frac{1}{2}at^2 + v_0 t + x_0,$$

where x_0 and v_0 are the position and velocity at $t = 0$, respectively.

4. Rotation with constant angular velocity ω (frequency $\nu = \frac{\omega}{2\pi}$) along a circle of radius R:

- polar coordinates

$$r = R, \qquad \theta = \omega t + \theta_0,$$

- Cartesian coordinates

$$x = R\cos(\omega t + \theta_0)\,, \qquad y = R\sin(\omega t + \theta_0)\,,$$

- linear velocity

$$v = R\omega,$$

- acceleration

$$a = R\omega^2,$$

where θ_0 is the value of θ at $t = 0$.

5. Rotation with constant angular acceleration α:

$$\omega = \alpha t + \omega_0, \qquad \theta = \tfrac{1}{2}\alpha t^2 + \omega_0 t + \theta_0,$$

where ω is the angular velocity, θ is the angular coordinate, ω_0 is the angular velocity at $t = 0$, and θ_0 is the angular coordinate at $t = 0$.

Dynamics

1. Newton's second law:

$$m\mathbf{a} = \mathbf{F}.$$

2. For a sliding body, the friction force is $F_{\mathrm{rf}} = kN$, when N is the normal reaction force. It is oriented in the opposite sense of the motion.

3. Conserved quantities:

   - linear momentum

$$\mathbf{P} = \sum_i m_i \mathbf{v}_i,$$

   - angular momentum with respect to the origin

$$\mathbf{A}_O = \sum_i m_i \mathbf{r}_i \times \dot{\mathbf{r}}_i,$$

   - angular momentum with respect to an arbitrary point P

$$\mathbf{A}_P = \sum_i m_i \left(\mathbf{r}_i - \mathbf{r}_P\right) \times \dot{\mathbf{r}}_i,$$

   - total energy

$$E = U(x_1, x_2, \ldots) + \sum_i \tfrac{1}{2} m_i v_i^2,$$

   where U is the potential energy.

4. A conservative force $\mathbf{F}$ and the corresponding potential energy U are related by

$$\mathbf{F} = -\boldsymbol{\nabla} U.$$

5. The potential energy and force for a spring of modulus k, and unperturbed length L_0 are

$$U = \tfrac{1}{2}k\left(L - L_0\right)^2, \qquad F = -k\left(L - L_0\right),$$

where L is the current length of the spring. The direction of $\mathbf{F}$ is such that it tries to bring the spring back to its unperturbed configuration.

6. The potential energy U and force F for a particle of mass m located at a height H, in the Earth's gravitational field are

- locally:

$$U = -mgH, \qquad \mathbf{F} = m\mathbf{g},$$

- globally:

$$U = -\frac{GM_{\mathrm{E}}m}{R_{\mathrm{E}} + H}, \qquad \mathbf{F} = -\frac{GM_{\mathrm{E}}m}{\left(R_{\mathrm{E}} + H\right)^2}\mathbf{e}_r,$$

where G is the gravitational constant, M_{E} is the Earth's mass, R_{E} is the Earth's radius, $\mathbf{e}_r$ is a radial unit vector. $G = 6.7\times10^{-11}\,\mathrm{m}^3\,\mathrm{kg}^{-1}\,\mathrm{s}^{-2}$, $M_{\mathrm{E}} = 6.0\times10^{24}\,\mathrm{kg}$, and $R_{\mathrm{E}} = 6.4\times10^6\,\mathrm{m}$.

- The angular velocity of a body rotating along a circular orbit around a much heavier body of mass M is

$$\omega = \sqrt{\frac{GM}{R^3}}$$

where R is the orbit radius.

Oscillations

The equation of a forced linear pendulum with small amplitude is

$$\ddot{\phi} + 2c\dot{\phi} + \omega^2\phi = F_0\cos(\Omega_0 t),$$

where c is the friction coefficient, $\omega^2 = \frac{L}{g}$ is the natural frequency of the pendulum, L is the length of the pendulum, F_0 and Ω_0 are the amplitude and frequency of the external forcing.

Hamiltonian mechanics

1. The Hamiltonian equations are

$$\dot{x}_j = \frac{\partial H}{\partial p_j}, \qquad \dot{p}_j = -\frac{\partial H}{\partial x_j}, \qquad 1 \le j \le n.$$

2. The Poisson brackets of functions

$$\{F, G\} = \sum_{i=1}^{n} \left(\frac{\partial F}{\partial p_i} \frac{\partial G}{\partial x_i} - \frac{\partial F}{\partial x_i} \frac{\partial G}{\partial p_i} \right).$$

3. A transformation

$$x_i' = x_i'(x_1, \ldots, x_n, p_1, \ldots, p_n), \qquad p_i' = p_i'(x_1, \ldots, x_n, p_1, \ldots, p_n),$$

is canonical if and only if

$$\{x_i', p_k'\} = -\delta_{ik}, \qquad \{x_i', x_k'\} = 0, \qquad \{p_i', p_k'\} = 0.$$

4. The Lagrangian equations are

$$\frac{\mathrm{d}}{\mathrm{d}t}\left(\frac{\partial L}{\partial \dot{x}_j}\right) - \frac{\partial L}{\partial x_j} = 0, \qquad 1 \le j \le n.$$

Stability of dynamical systems

Let $\mathbf{x}_{\mathrm{F}}$ be a fixed point of a dynamical system $\dot{\mathbf{x}} = \mathbf{f}(\mathbf{x})$, where

$$\mathbf{x} = \begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ x_k \end{bmatrix}, \qquad \mathbf{f} = \begin{bmatrix} f_1(x_1, \ldots, x_k) \\ f_2(x_1, \ldots, x_k) \\ \vdots \\ f_k(x_1, \ldots, x_k) \end{bmatrix}.$$

Then,

$$\mathbf{J} = \begin{bmatrix} \frac{\partial f_1}{\partial x_1} & \frac{\partial f_1}{\partial x_2} & \cdots & \frac{\partial f_1}{\partial x_k} \\ \frac{\partial f_2}{\partial x_1} & \frac{\partial f_2}{\partial x_2} & \cdots & \frac{\partial f_2}{\partial x_k} \\ \vdots & \vdots & \ddots & \vdots \\ \frac{\partial f_k}{\partial x_1} & \frac{\partial f_k}{\partial x_2} & \cdots & \frac{\partial f_k}{\partial x_k} \end{bmatrix},$$

is the Jacobian matrix of the system at $\mathbf{x}_{\mathrm{F}}$, with eigenvalues λ_1, λ_2, …, λ_k.

- If $\Re(\lambda_j) < 0$ for all j then $\mathbf{x}_\mathrm{F}$ is asymptotically stable.
- If $\Re(\lambda_j) > 0$ for some j then $\mathbf{x}_\mathrm{F}$ is unstable.
- If $\Re(\lambda_j) < 0$ for some j, and $\Re(\lambda_j) = 0$ for the remaining j, then the test is inconclusive.