# Friction

MS4414 Theoretical Mechanics

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Friction is a force opposing or preventing motion. The frictional force $F$ takes a maximum value of $kN$ where $N$ is the normal reaction force and $k$ is the friction coefficient. The normal reaction force is the force the surface the particle is resting on exerts on it to hold it up. Friction acts in the opposite direction to motion or, if the object is stationary, in the opposite direction to applied forces.

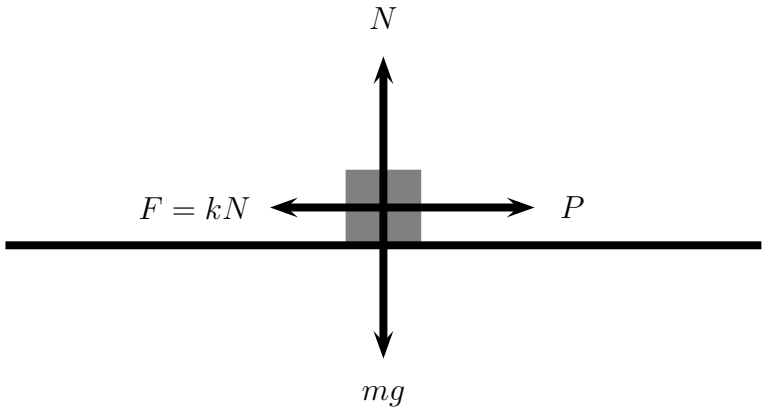
## 1 Friction on a flat surface

A particle on a flat surface is being pulled with force $P$. The particle has mass $m$ and weight of the particle is $mg$. The normal reaction is $N = mg$. The maximum frictional force $F_{\text{max}} = kN$.

If $P < F_{\text{max}}$ then $F = P$ and the particle remains stationary.

If $P > F_{\text{max}}$ then $F = F_{\text{max}}$ and the particle accelerates in the direction of $P$ with acceleration $a$ given by Newton's second law

$$P - F_{\text{max}} = ma$$




**Example** Shove Ha'penny.

**Example** Interpreting tire marks.

**Example** Formula one cars turning corners.

## 2 Friction on inclined planes

Sliding on a slope. The weight of the particle $mg$ directed downwards must be resolved into components parallel to the slope ($mg\sin\theta$) and perpendicular to the slope ($mg\cos\theta$).

The component perpendicular to the slope ($mg\cos\theta$) will be balanced by the normal reaction ($N = mg\sin\theta$).

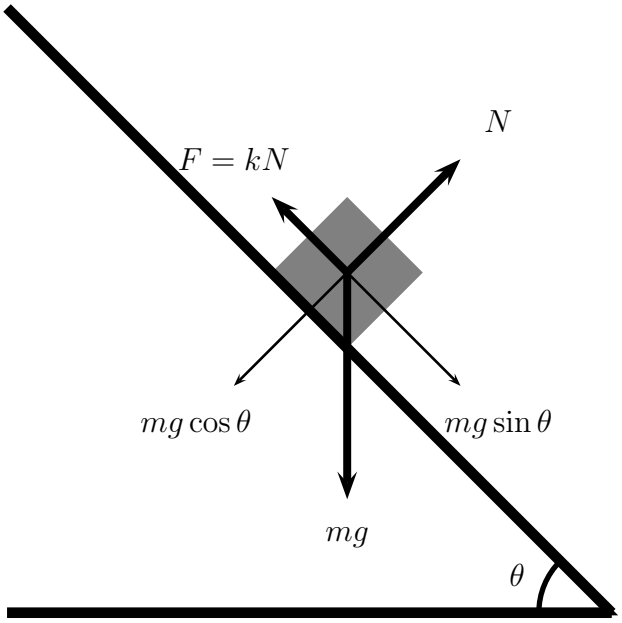
The maximum force friction can exert on the particle is $F_{\text{max}} = kN$.

If the component of the particle's weight down the slope is smaller than this maximum friction force $mg\sin\theta < F_{\text{max}}$ then $F$ takes the value $F = mg\sin\theta$ and the

particle remains stationary.

If the component of the particle's weight down the slope is larger than the maximum friction force then the friction force takes its maximum value $F_{\text{max}}$. The net force downhill is therefore $mg\sin\theta - F_{\text{max}}$. By Newton's second law the acceleration $a$ down-slope is given by

$$mg\sin\theta - F_{\text{max}} = ma$$




**Example** Hanging a spoon from your nose.