Dynamical Systems

Question 1 *(2005 paper.)* Find and examine the fixed points of

$$\dot{\phi} = \psi, \qquad \dot{\psi} = \phi^2 + \phi\psi - 1$$

Question 2 *(2006 paper.)* Find and examine the fixed points of

$$\dot{\phi} = -\psi, \qquad \dot{\psi} = \phi^2 - \phi\psi - 1$$

Question 3 *(2007 paper.)* Find and examine the fixed points of

$$\dot{\phi} = \phi + \psi^2 + \psi, \qquad \dot{\psi} = \phi + \psi^2 + 1$$

Question 4 *(2009 paper.)* Consider the following dynamical system.

$$\dot{\phi} = (\phi + 1)\psi, \qquad \dot{\psi} = \phi(\psi - 1)$$

(a) Show that the fixed points of this dynamical system are exactly $(0,0)$ and $(-1,1)$.

(b) Examine the stability of each fixed point.