The Astrolabe

Mathematical Modelling Summer School 2009

25th May 2009

Astrolabe

Used in medieval times to

- Showing the night sky will look like
- Telling the time
- Calculating rising and setting times of celestial objects

Placing the planets

The sun and planets lie in a plane called the *ecliptic*, denoted by the the numerals I to XII on the star plate. (These numerals give the position of the sun at the start of each month.)

Sun	☉	VII
Mercury	☿	VII–VI
Venus	♀	VI – V
Mars	♂	VI–V
Jupiter	♃	III
Saturn	♄	X-IX
Moon (dark)	☾	IX-VIII

Showing the night sky

To show the night sky at 2200GMT(=2300IST) line up todays date on the star plate with 22 on the overlay. Visible objects lie inside the thick distorted circle.

Visible objects include

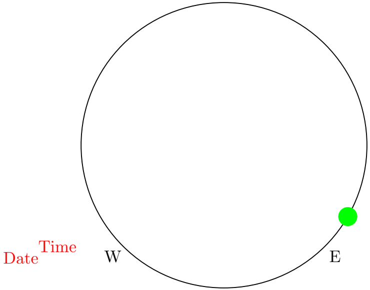
- Saturn ♄
- Cassiopea
- Cygnus
- Lyra

Rising and setting time

To calculate the time at which an object will rise (set) rotate the star plate until the object is on the eastern (western) side. The rise time (GMT) on the overlay corresponds to today's date on the star plate. Add one hour for IST.

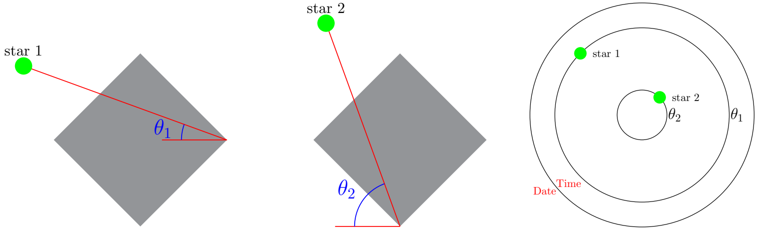
Rise times

Rise time		GMT	IST
♃	Jupiter	2300	0000
♀	Venus	0200	0300
♂	Mars	0200	0300
☿	Mercury	0300	0400
☉	Sun	0400	0500



Telling the time

To tell the time measure the angular altitude of two stars. Rotate the star plate until the two stars are at the correct altitudes on the overlay. As before today's date on the star plate gives the time on the overlay.



Planetary orbits

Assume planets move in regular circular orbits around the sun in the ecliptic plane. (This takes us up to Copernicus, but ignores improvements by Kepler, Newton and Einstein.) Orbit has fixed radius r, and angle in plane increases linearly with time $\theta - \theta_0 = \omega t$. In cartesian coordinates

$$x = r \cos \omega t \tag{1}$$

$$y = r \sin \omega t \tag{2}$$

Or more elegantly

$$x = \mathcal{R} z \tag{3}$$

$$y = \mathcal{I} z \tag{4}$$

$$z = r e^{i \omega t} \tag{5}$$

The Maya and 584

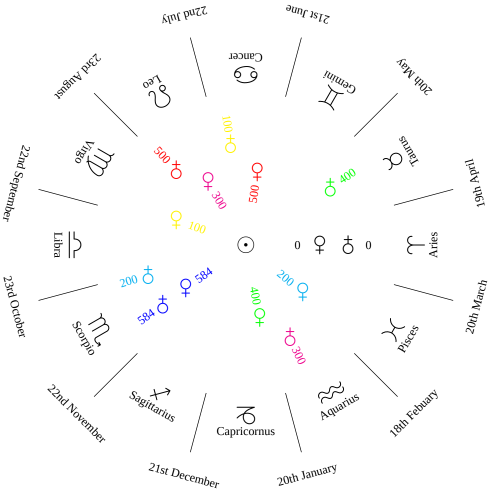
Dresden codex.

Orbital parameters

$r_{♁}$	0.723 au
$r_{♀}$	1.000 au
$r_{♂}$	1.523 au
$T_{♀}$	224.7 days
$T_{♁}$	365.2 days
$T_{♂}$	687.0 days

$$\omega = 2\pi/T$$

Earth and Venus



Epicycles

Things are simpler with a heliocentric solar system

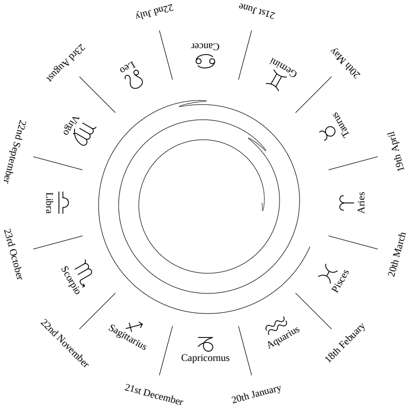
Motion of mars as seen from Earth

$$\theta_{\text{♁}\text{♂}} = \arg\left(z_{\text{♁}} - z_{\text{♂}}\right) \tag{6}$$

Alternatively

$$\theta_{\text{♁}\text{♂}} = \arctan\frac{y_{\text{♁}} - y_{\text{♂}}}{x_{\text{♁}} - x_{\text{♂}}} \tag{7}$$

Retrograde motion



From time to time Mars appears to move backwards (retrograde motion). Modelled by epicycles: a circle within a circle.

An astronomical insult

Helen	Monsieur Paroles, you were born under a charitable star
Paroles	Under Mars, I. . . . When he was predominant.
Helen	When he was *retrograde* I think rather.
Paroles	Why think you so?
Helen	You go so much backwards when you fight.

Alls Well That Ends Well. William Shakespeare.