Fourier Series, Exercises

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Question 1 A periodic function with period 2π is defined by

$$\begin{aligned} f(x) &= 2 && 0 < x < \pi \\ f(x) &= -2 && \pi \leq x < 2\pi \end{aligned}$$

Sketch the graph of $f(x)$ for $-2\pi < x < 2\pi$ and obtain a Fourier series expansion of this function. Use the series to find an expression for π. *(2003 paper.)*

Question 2 Find the Fourier series of period 2π of the function

$$\begin{aligned} f(x) &= 1 + x && 0 < x < \pi \\ f(x) &= 1 - x && -\pi < x \leq 0 \\ f(x) &= f(x + 2\pi) \end{aligned}$$

Use your answer to find an expression for π^2. *(2004 paper.)*

Question 3 Find the Fourier series of period 2π of the function

$$\begin{aligned} f(x) &= -x && -\pi < x < \pi \\ f(x) &= f(x + 2\pi) \end{aligned}$$

Sketch a graph of $f(x)$ and use the series to find an expression for π. *(2005 paper.)*

Question 4 Find the Fourier series of period 2π of the function

$$\begin{aligned} f(x) &= -x && 0 < x < \pi \\ f(x) &= x && -\pi < x \leq 0 \\ f(x) &= f(x + 2\pi) \end{aligned}$$

Use your answer to find an expression for π^2. *(2006 paper.)*

Question 5 The function $f(x)$ is periodic with period 2π is defined over the interval $[-\pi, \pi]$ by

$$\begin{aligned} f(x) &= -3 && -\pi < x < 0 \\ f(x) &= 0 && 0 \leq x < \pi \end{aligned}$$

Find its Fourier series. *(2007 paper.)*

Question 6 Find the Fourier series of period 2π of the function

$$\begin{aligned} f(x) &= |x| && -\pi < x < \pi \\ f(x) &= f(x + 2\pi) \end{aligned}$$

By taking $x = 0$ deduce an expression for π written in terms of a series. *(2008 paper.)*