Laplace Transforms, Exercises

Sofiane Soussi
and William Lee
william.lee@ul.ie
http://www.ul.ie/wlee/ma4005.html

November 15, 2010

Exercises

1 Exercises

Question 1 Calculate the Laplace transform of the following functions directly from the definig formula.

(i) $2t^2$

(ii) $3\sin(2t)$

(iii) $3e^{-t}$

Question 2 Using the tables write down the transforms of the following functions/

(i) $t^2 - 3t + 4$

(ii) $2\cos t + 3\sin 2t$

(iii) $4e^t - e^{-2t}$

(iv) $(t^2 - 3)^2$

(v) $2\cos^2 t$

(vi) $e^{-t}(e^{2t} + 2e^{-t})^2$

(vii) $4\sin t \cos t$

(viii) $1 - 4\sin t$

Question 3   Use the tables to find the inverse transforms of the following functions.

(i) $\frac{1}{s^2}$

(ii) $\frac{2}{s-1}$

(iii) $\frac{3s}{s^2+4}$

(iv) $\frac{(s+2)^2}{s^4}$

(v) $\frac{4}{s^2-1}$

(vi) $\frac{s+6}{s^2+9}$

(vii) $\frac{3}{s-2} + \frac{2}{s^2+2}$

(viii) $\frac{1-3s}{s^2+2}$

Question 4   Using the first shifting theorem, determine the transform of the following functions.

(i) $e^t \sin 4t$

(ii) te^{-t}

(iii) $e^t \cosh t$

Question 5   Sketch these functions.

(i) $g(t) = \mathcal{U}_1(t)t^2$

(ii) $h(t) = \mathcal{U}_2(t)e^{-t}$

(iii) $k(t) = \mathcal{U}_\pi(t) \cos \frac{t}{2}$

In each case, find the Laplace transform of the function.

Question 6   Find the inverse transform of the following functions.

(i) $\frac{e^{-s}}{s^2+1}$

(ii) $\frac{e^{-2s}}{s^3}$

(iii) $\frac{e^{-3s}}{(s-1)^2+1}$

Question 7   Find the inverse Laplace transform of the following functions.

(i) $\frac{1}{s^2+2s+1}$

(ii) $\frac{2}{s^3-s^2+s-1}$

(iii) $\frac{2s+4}{(s+1)(s^2-2)}$

(iv) $\frac{1}{s((s-1)^2+4)}$

(v) $\frac{2s-1}{4s^2+3s-1}$

(vi) $\frac{3s^2+7s+10}{s^3+2s^2+5s}$

(vii) $\frac{e^{-s}(s^2+5)}{s^3+2s^2-s-2}$

(viii) $\frac{4s-2}{s^3+6s^2+11s+6}$

Question 8   Use Laplace transforms to find the solutions to the following first order differential equations.

(i) $y' + 4y = 2\sin 2t$, $y(0) = 1$.

(ii) $2y' - y = 2e^{2t}$, $y(0) = 3$.

(iii) $y' + y = 2t^2 - 1$, $y(0) = -1$.

Question 9   Solve these second order differential equations by applying the Laplace transform.

(i) $6y'' - 5y' - y = 0$, $y(0) = 2$, $y'(0) = 1$.

(ii) $4y'' - 12y' + 9y = 0$, $y(0) = 0$, $y'(0) = 1$.

(iii) $y'' + 4y = \sin 2t$, $y(0) = 1$, $y'(0) = 0$.