

Differential Equations, Exercises

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Exercises

Question 1 Show that

$$y = e^x + ce^{-4x}$$

is a solution of

$$\frac{dy}{dx} + 4y = 5e^x.$$

If $y(0) = 2$, what is the particular solution?

Question 2 Implicit solutions : sometimes solutions cannot be expressed explicitly. Show that

$$y^2 - x^2 + 4xy = c$$

is the general solution of

$$(2x + y)dy = (x - 2y)dx.$$

What is the particular solution if $y = 1$ when $x = 1$?

Question 3 Find the general solution of the following differential equations and the particular solution for the given initial condition.

(i)

$$y' + xy = xy^2, \quad y(0) = 2.$$

(ii)

$$\frac{dy}{dx} + \frac{x(2y - 3)}{x^2 + 1} = 0, \quad y(0) = -1.$$

(iii)

$$\ln(y) \frac{dy}{dx} + \frac{y}{x} = 0, \quad y(1) = e^2.$$

(iv)

$$y' = y \tan(2x), \quad y(0) = 1.$$

Question 4 Solve the following linear equations.

(i)

$$x \frac{dy}{dx} - 2y = x^3 e^x$$

(ii)

$$\frac{dy}{dx} - y = \sin(x)$$

(iii)

$$\frac{dy}{dx} - y = (x + 1)^2, \quad y(0) = 0$$

(iv)

$$x \frac{dy}{dx} - 3y = x^4(e^x - \cos(x)) - 2x^2$$

Question 5

- (i) The only force resisting an object falling freely in a vacuum is gravity. Therefore, by Newton's second law of motion, the fall is governed by the equation

$$m \frac{dv}{dt} = -mg,$$

where m is the mass of the body and v is the velocity. Solve this equation for $v(t)$ and then integrate $v(t)$ to get $s(t)$, the distance moved. If the object falls from a height of 100 meters, how long will it take before it strikes the ground? (Take $g = 9.81m/s^2$)

- (ii) Outside a vacuum, the object also encounters air resistance. We may assume that the air resistance is proportional to the speed of the object. The fall of the object is then governed by the equation:

$$m \frac{dv}{dt} = -kv - mg,$$

where k is some constant. Solve the above equation for $v(t)$, assuming that the initial velocity of the object is zero. Suppose the object has a mass of 10 grams and is initially at a height of 100 metres. Take $k = 0.05$. How long, approximately, does it take for the object to reach the ground?

Question 6 For each of the following homogeneous equations, write down

- (a) the auxiliary equation.
- (b) the roots of the auxiliary equation.
- (c) the general solution for the differential equation.

(i) $y'' - 3y = 0$

(ii) $y'' - 6y' + 25y = 0$

(iii) $y'' + 10y' + 24y = 0$

(iv) $3y'' - 4y' = 0$

(v) $9y'' - 6y' + y = 0$

(vi) $y'' + 2y' + 3y = 0$

Question 7 Find a particular integral for each of the following differential equations.

(i) $y'' - y = t(t + 1)$

(ii) $y'' - y = e^{2t}$

(iii) $y'' - y = \sin(3t)$

(iv) $y'' - y = e^{-t}$

(v) $y'' - y = 6e^{-2t} - 2t + 3$

(vi) $y'' - y = -2e^{-t}$

(vii) $y'' + y = t^3 + 1$

(viii) $y'' + y = e^t - e^{2t}$

(ix) $y'' + y = \cos(2t)$

(x) $y'' + y = \sin(t)$

(xi) $y'' + 2y' + 2y = \cosh(t)$