Integration, Exercises

Sofiane Soussi
and William Lee
william.lee@ul.ie
http://www.ul.ie/wlee/ma4005.html

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Question 1 Evaluate the following integrals:

(i) $\int (3x^2 + 2x)dx$

(ii) $\int (\sin x + \cos x)dx$

(iii) $\int (e^x + e^{-x})dx$

(iv) $\int (x + 2)^2 dx$

Question 2 Use the method of integration by parts to evaluate:

(i) $\int x \sin x dx$

(ii) $\int e^x \cos x dx$

(iii) $\int x e^x dx$

(iv) $\int x^2 \sin(2x) dx$

(v) $\int x \ln x dx$

(vi) $\int \sqrt{x} \ln x dx$

Question 3 By choosing a suitable substitution, evaluate the following integrals.

(i) $\int t(t^2-1)^3 dt$

(ii) $\int \frac{(\sqrt{u}+3)^4}{\sqrt{u}} du$

(iii) $\int \sqrt{1-x^2} dx$

(iv) $\int \sqrt{1+x^2} dx$

(v) $\int \frac{dx}{\sqrt{x}(1+\sqrt{x})^2}$

(vi) $\int e^x \cos(e^x+2) dx$

Question 4 Integrate the following by making use of the 't' substitution.

(i) $\int \frac{dx}{\sin x + 2\cos x}$

(ii) $\int \frac{\sin x}{\sin^2 x - 2\cos x} dx$

(iii) $\int \frac{dx}{1+2\sin x}$

Question 5 For each of the following complete the square on the denominator and then evaluate the integral.

(i) $\int \frac{dt}{t^2+4t+5}$

(ii) $\int \frac{dt}{2t^2+3t+2}$

(iii) $\int \frac{2t dt}{3t^2+6t+9}$

Question 6 In each of the following express the integrand as a sum of partial fractions. Then do the actual integral.

(i) $\int \frac{dx}{x^2-3x+2}$

(ii) $\int \frac{dx}{x^3-x^2-x+1}$

(iii) $\int \frac{2x+1}{x^3-1} dx$

Question 7 Evaluate the following integrals using any method that is convenient.

(i) $\int \frac{x}{(x+1)^3} dx$

(ii) $\int \frac{4\sin x - 3\cos x}{2\sin x + \cos x} dx$

(iii) $\int \frac{\cos^8 x}{\sin^3 2x} dx$

(iv) $\int \frac{e^{2t}}{\sqrt{1+e^{2t}}} dt$

(v) $\int \frac{x}{x^3 - 5x^2 + 8x - 4} dx$

(vi) $\int \frac{x^2}{\sqrt{x^3 - 3}} dx$

(vii) $\int (1 - x^2)^{\frac{3}{2}} dx$

(viii) $\int \cos x \sin^5 x dx$

(ix) $\int \sin(3x)\sin(8x) dx$

Question 8 For each of the following functions sketch the graph and then find the area of the surface enclosed between the curve of the function and the x axis for $x \in [a, b]$.

(i) $f_1(x) = x^2 - 4$, $a = -3$, $b = 1$.

(ii) $f_2(x) = \ln(x)$, $a = \frac{1}{2}$, $b = 2$.

Question 9 Find the the length of the curve of the function f, for $x \in [0, \ln(2)]$, where $f(x) = \cosh(x)$.

We recall that :

$$\cosh(x) = \frac{e^x + e^{-x}}{2}, \quad \sinh(x) = \frac{e^x - e^{-x}}{2}$$
$$\cosh'(x) = \sinh'(x), \quad \sinh'(x) = \cosh'(x).$$

Question 10 Let f_1 and f_2 be defined by:

$$f_1(x) = \sqrt{x}, \qquad f_2(x) = x^2.$$

(i) Sketch the curves of the functions f_1 and f_2 .

(ii) Find the volume obtained by rotating the surface enclosed between the curves of f_1 and f_2 for $x \in [0, 1]$ about the x axis.

Question 11 For each of the two following cases answer the questions (i), (ii), (iii).

(a) $f(x) = x^2$, $g(x) = 1$.

(b) $f(x) = x^2 - 2x$, $g(x) = x$.

(i) Sketch the curves of f and g.

(ii) Find the area of the surface enclosed between the two curves for $x \in [0, 1]$ in case (a) and for $x \in [0, 3]$ in case (b).

(iii) Find the volume obtained by rotating the previous surfaces about the x axis.

Question 12 For each of the following cases, sketch the curves of f and g, then find the coordinates $(\bar{x}, \bar{y})$ of the centroid of the surface enclosed between the curves of f and g for $x \in [a, b]$.

(a) $f(x) = e^x$, $g(x) = e$, $a = 0$, $b = 1$.

(b) $f(x) = 8 - x^2$, $g(x) = 2x$, $a = 0$, $b = 2$.

Question 13

(i) Sketch the curves of the functions f and g for $x \in [-1, 2]$, where

$$f(x) = e^x, \qquad g(x) = e.$$

(ii) Find the moment of inertia I_y about the y axis of the surface enclosed by the two curves for $x \in [0, 1]$.

(iii) Find the corresponding radius of gyration k_y.

(iv) Find the moment of inertia I_x about the x axis of the same surface.

(v) Find the corresponding radius of gyration k_x.

(vi) Redefine the boundary of the area using two functions h and k of the form:

$$x = h(y), \qquad x = k(y).$$

(vii) Find the moment of inertia J_x about the x axis of the same surface using the new boundary.

(viii) Find the corresponding radius of gyration l_x.

(ix) Compare I_x and k_x with J_x and l_x.

(x) Find the moment of inertia I_0 about the origin.

(xi) Find the Radius of gyration k_0 about the origin.