# Partial Differentiation, Exercises

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**Question 1** Find the first partial derivatives of :

(i) $f_1(x, y) = x^2 + y^3$

(ii) $f_2(x, y) = x^3y^5$

(iii) $f_3(x, y) = (2xy^2 - 4y)e^x$

(iv) $f_4(x, y) = \sin(2x + 5y)$

(v) $f_5(x, y) = 5xy^3 \cos(x + y)$

(vi) $f_6(x, y) = \frac{x}{y^2} - \frac{y}{x^2}$

(vii) $f_7(x, y) = \ln(x^2 + y)$

(viii) $f_8(x, y, z) = xy + yz + xz$

(ix) $f_9(x, y, z, w) = \frac{w^2}{xy^2z^3}$

**Question 2** Find the second partial derivatives of the previous functions.

**Question 3** A cylindrical hole of diameter 6 inches and height 4 inches is to be cut in a block of wood by a process in which the maximum error in diameter is 0.05 inch and in height is 0.01 inch. What is the largest possible error in the volume of the cavity?

**Question 4** The breaking weight $W$ of a cantilever beam is given by the formula

$$Wl = Kbd^2,$$

where $b$ is the breadth, $l$ the length, $d$ the depth, and $K$ a constant depending on the material of the beam.

If the length is increased by 1% and the breadth by 5%, by how much should the depth be altered to keep the breaking weight unchanged?

**Question 5** A triangle $ABC$ is being transformed so that the angle A changes at a uniform rate from $0°$ to $90°$ in 10 seconds while side $AB$ increases by $1\,\mathrm{cm\,s^{-1}}$ and side $AC$ decreases by $1\,\mathrm{cm\,s^{-1}}$. If at the time of observation, $A = 60°$, $AC = 16\,\mathrm{cm}$, and $AB = 10\,\mathrm{cm}$, find

(i) how fast is $BC$ changing?

(ii) how fast is the area of the triangle changing?

**Question 6** The radius of a cylinder increases at the rate of $2\,\mathrm{cm\,s^{-1}}$ and the height $h$ increases at $3\,\mathrm{cm\,s^{-1}}$. Find the rate at which the volume is increasing when $r = 10\,\mathrm{cm}$ and $h = 20\,\mathrm{cm}$.

**Question 7** (a) Find all partial derivatives of order 2 of the following functions: $f(x, y) = ye^{3xy}\cos(xy)$; $g(x, y) = \frac{2x-3y}{x^2-2y}$. (b) In an ideal gas, the pressure $P$, the volume $V$, the temperature $T$, and the amount of gas $n$ (in moles) satisfy the following formula: $PV = nRT$ where $R$ is the ideal gas constant. We consider a fixed quantity of gas $n_0$, enclosed in a box of volume $V_0$, maintained at a temperature $T_0$. Starting from that initial state, we deform slightly the box so that it's volume reduces by $\delta V$, which is supposed to be small (the new volume of the gas is $V_0 - \delta V$, and at the same time we heat the box so that the temperature of the box is raised by $\delta T$ (the new temperature is $T_0 + \delta T$). (i) Write the total differential of $P$ in terms of $n$, $T$ and $V$. (ii) Supposing that all parameters have changed very slightly , find an approximation of the pressure $P$ of the gas in the new state in terms of $R$, $n_0$, $P_0$, $V_0$, $T_0$, $\delta V$ and $\delta T$. *Exam 2008-9*