15 Oct. 2010

1. Find the first partial derivatives of the following functions.
   (a) $f(x,y) = e^{2x+y} + x^3 y$ 5%
   (b) $f(x,y) = x \ln y$ 5%
   (c) $f(x,y) = \dfrac{1}{(4x + 2y^3)}$ 5%
   (d) $f(x,y) = \cos(x/y)$ 5%
2. Find the second partial derivatives of the following functions.
   (a) $f(x,y) = \sin(x + y^2)$ 10%
   (b) $f(x,y) = (x - y)^4$ 10%
3. The equation of an ideal gas is $PV = nRT$ where P is pressure, V is volume, n is amount, R is the ideal gas constant and T is temperature. A fixed amount of gas is held in a container which is being heated at a rate of $1\,\mathrm{K\,s^{-1}}$ and contracted at a rate of $0.01\,\mathrm{m^3\,s^{-1}}$. Given that the amount of gas in the container is $5\,\mathrm{mol}$ and currently the volume is $0.2\,\mathrm{m^3}$ and the temperature is $300\,\mathrm{K}$, what is the rate of change of the pressure? ($R = 8.31\,\mathrm{J\,K^{-1}\,mol^{-1}}$.) 20%
4. For each of the following complete the square on the denominator and then evaluate the integral.
   (a) $\int \frac{dt}{t^2+4t-1}$ 10%
   (b) $\int \frac{dt}{3t^2+6t+6}$ 10%
5. Integrate the following by making use of the 't' substitution.
   (a) $\int \frac{dx}{\sin x + 5\cos x}$ 20%