FACULTY OF SCIENCE AND ENGINEERING

DEPARTMENT OF MATHEMATICS & STATISTICS

END OF SEMESTER ASSESSMENT PAPER

MODULE CODE: MA 4005

SEMESTER: Autumn 2013

MODULE TITLE: Engineering Maths T1

DURATION OF EXAMINATION: 2hrs 30mins

LECTURER: Dr. William Lee

PERCENTAGE OF TOTAL MARKS: 80%

INSTRUCTIONS TO CANDIDATES:
Answer any 5 questions. All questions carry equal marks. Full marks for correct answers to any 5 questions.
Open book exam.

1. Find all first order and second order partial derivatives of the following functions: 20%

   (a) $f_1(x, y) = x^2 + y^4 + x^3y^2$ 5%

   (b) $f_2(x, y) = x \sin(x^2 + y^2)$ 5%

   (c) $f_3(x, y, z) = \exp(x + y) \cos(y + z)$ 10%

2. The Tsiolkovsky rocket equation is 20%

$$v = v_\mathrm{e} \ln\left(\frac{m_\mathrm{i}}{m_\mathrm{f}}\right)$$

   where v is the final velocity of a rocket initially at rest, after its mass has decreased from m_i to m_f due to the ejection of propellant at exhaust velocity v_e. These quantities take the values $v_\mathrm{e} = 100 \pm 10\,\mathrm{km\,s^{-1}}$, $m_\mathrm{i} = 9000 \pm 100\,\mathrm{kg}$ and $m_\mathrm{f} = 1000 \pm 50\,\mathrm{kg}$.

   (a) Write the total differential of v in terms of v_e, m_i, m_f. 5%

   (b) Write an expression for the maximum error in v in terms of v_e, m_i, m_f and their uncertainties, δv_e, δm_i, δm_f (assuming those uncertainties to be small). 5%

   (c) Calculate the numerical value of the maximum error in v using the values given above. 10%

3. Calculate the following integrals. 20%

   (a) $\int (x^5 + 2\cos(2x) + e^{-3x})\,\mathrm{d}x$ 5%

   (b) $\int x^2 \exp(x^3)\,\mathrm{d}x$ 5%

   (c) $\int x \cos(2x)\,\mathrm{d}x$ 5%

   (d) $\int \frac{x}{x^2+6x+5}\,\mathrm{d}x$ 5%

4. 20%

   (a) Find the area under the curve $y = x^3$ and above the x-axis between $x = 0$ and $x = 2$. 6%

   (b) Find the area between the curves $y = x^3$ and $y = x^2$ between $x = 0$ and $x = 2$. 6%

   (c) Find the volume generated when the previously defined area—between the curves $y = x^3$ and $y = x^2$ between $x = 0$ and $x = 2$—is rotated about the x axis. 8%

5. Find the general solution of the differential equations [20%]

   (a) $y' + 2y = 0$ [5%]

   (b) $y' - x^3 y = 0$ [5%]

   (c) $y'' + 5y' + 4y = 5x$ [10%]

6. [20%]

   (a) Use integration by parts to show that the Laplace transforms of y' and y'' are $sY(s) - y(0)$ and $s^2Y(s) - sy(0) - y'(0)$ respectively, where Y is the Lapalace transform of y. [10%]

   (b) Use the Laplace transform to find the solution of the differential equation [10%]

$$y'' - 5y' + 6y = 3, \qquad y(0) = 1, \quad y'(0) = -1.$$