UNIVERSITY of LIMERICK

OLLSCOIL LUIMNIGH

FACULTY OF SCIENCE AND ENGINEERING

DEPARTMENT OF MATHEMATICS & STATISTICS

END OF SEMESTER ASSESSMENT PAPER

MODULE CODE: MA 4005

SEMESTER: Autumn 2011

MODULE TITLE: Engineering Maths T1

DURATION OF EXAMINATION: 2hrs 30mins

LECTURER: Dr. William Lee

PERCENTAGE OF TOTAL MARKS: 80%

INSTRUCTIONS TO CANDIDATES:
Answer any 5 questions. All questions carry equal marks.
Full marks for correct answers to any 5 questions.
Open book exam.

1. Find all first order and second order partial derivatives of the following functions: 20%

   (a) $f_1(x, y) = 5x^2 + 4y^3 + xy$ 5%

   (b) $f_2(x, y) = (x + y)\sin^2(x + y)$ 5%

   (c) $f_3(x, y, z) = \exp\left(x^2 + xy + y^2 - yz + z^2\right)$ 10%

2. The rise velocity, v of a small gas bubble in a liquid is given by 20%

$$v = \frac{2r^2\rho g}{9\mu}$$

   where r is the radius of the bubble, $\rho = 10^3\,\mathrm{kg\,m^{-3}}$ is difference in density between the liquid and the gas, $g = 9.81\,\mathrm{m\,s^{-2}}$ is the acceleration due to gravity, and μ is the viscosity of the liquid.

   (a) Rearrange the equation to get an equation for the viscosity μ. 5%

   (b) Write the total differential of μ treating ρ and g as constants and r and v as variables. 5%

   (c) Write an expression for the maximum error in μ in terms of r, v, and their uncertainties, δr and δv (assuming those uncertainties to be small). 5%

   (d) Measurements of r and v give $r = 1\,\mathrm{mm} \pm 0.1\,\mathrm{mm}$ and $v = 10\,\mathrm{mm\,s^{-1}} \pm 0.5\,\mathrm{mm\,s^{-1}}$. Calculate the estimated value of μ and its maximum error. (SI units of viscosity are Pa s.) 5%

3. Calculate the following indefinite integrals. 20%

   (a) $\int \left(4x^3 - e^x + \frac{2}{x}\right)\,\mathrm{d}x$ 5%

   (b) $\int \frac{x^3}{x^4+1}\,\mathrm{d}x$ 5%

   (c) $\int xe^{-2x}\,\mathrm{d}x$ 5%

   (d) $\int x\sqrt{x^2 - 1}\,\mathrm{d}x$ 5%

4. Consider the curve $y = \cosh(x)$ between $x = 0$ and $x = 2$. Use definite integrals to find: 20%

   (a) The length of the curve. 6%

   (b) Find the area between the curve and the x axis. 6%

   (c) Find the volume generated when this area is rotated about the x axis. 8%

5. Find the general solution of the differential equations (a prime, $'$, denotes differentiation with respect to x: $y' = \frac{\mathrm{d}y}{\mathrm{d}x}$) 20%

   (a) $y' - y = 0$ 5%

   (b) $y' + \cos(x)\, y = 0$ 5%

   (c) $2y'' + y' + 4y = 1$ 10%

6. Use Laplace transforms to solve the differential equations (a prime, $'$, denotes differentiation with respect to x: $y' = \frac{\mathrm{d}y}{\mathrm{d}x}$) 20%

   (a) $y' + 3y = 7, \quad y(0) = 1.$ 5%

   (b) $y' - 3y = 4, \quad y(0) = -1.$ 5%

   (c) $3y'' + 6y' + 5y = 0, \qquad y(0) = 1, \quad y'(0) = -1.$ 10%