UNIVERSITY *of* LIMERICK

OLLSCOIL LUIMNIGH

FACULTY OF SCIENCE AND ENGINEERING

DEPARTMENT OF MATHEMATICS & STATISTICS

END OF SEMESTER ASSESSMENT PAPER

MODULE CODE: MA 4005

SEMESTER: Autumn 2009

MODULE TITLE: Engineering Maths T1

DURATION OF EXAMINATION: 2hrs 30mins

LECTURER: Dr. S. Soussi

PERCENTAGE OF TOTAL MARKS: 80%

INSTRUCTIONS TO CANDIDATES:
Answer any 5 questions. All questions carry equal marks. Full marks for correct answers to any 5 questions.
Open book exam.

1. Find all partial derivatives of order 2 of the following functions: 20%

   (a) $f_1(x, y) = \sin(xy)$ 5%

   (b) $f_2(x, y) = xy^2 - 3x^2y$ 5%

   (c) $f_3(x, y) = (x + 2y)^2$ 5%

   (d) $f_4(x, y) = e^{xy}\cos(xy)$ 5%

2. In an ideal gas, the pressure P, the volume V, the temperature T, and the amount of gas n (in moles) satisfy the following formula: 20%

$$PV = nRT,$$

   where R is a constant called the gas constant.

   We consider a fixed quantity of gas n_0 enclosed in a box of volume V_0 maintained at a temperature T_0. Starting from that initial state, we deform slightly the box so that its volume is reduces by δV which is supposed to be small (the new volume is $V_0 - \delta V$), and at the same time, we heat the box so that the temperature of the gas is raised by δT (the new temperature is $T_0 + \delta T$).

   (a) Write the total differential of P in terms of n, T, V. 10%

   (b) Supposing that all parameters have changed very slightly, find an approximation of the pressure P of the gas in the new state in terms of R, n_0, P_0, V_0, T_0, δV and δT. 10%

3. (a) Find the area under the curve $y = e^{2x}$ and the x-axis between $x = 0$ and $x = 1$. 6%

   (b) Find the centroid of the previously defined area. 6%

   (c) Find the volume generated when the previously defined area is rotated about the x axis. 8%

4. Evaluate the definite integrals 20%

   (a) $\displaystyle\int_0^1 (x-1)^{10}dx$ 5%

   (b) $\displaystyle\int_1^5 \frac{2x+1}{x^2+x}dx$ 5%

   (c) $\displaystyle\int_2^3 \frac{dx}{x^2+2x+2}$ 5%

   (d) $\displaystyle\int_0^\pi e^x \sin(x)dx$ 5%

5. Find the general solution of the differential equations 20%

   (a) $y' - 2y = 0$ 10%

(b) $y' - 2y = \sin(x)$ 10%

6. 20%

(a) Calculate the Laplace transform of $f(t) = te^{-3t}$. 6%

(b) Use log tables to find the Laplace transform of the functions 6%

i. $f(t) = \cosh(t) - \sinh(t)$

ii. $f(t) = U_\pi(t)\cos(t - \pi)$

(c) Use the Laplace transform to find the solution of the boundary value problem 8%

$$y'' - 2y' + y = 2, \qquad y(0) = 1, \quad y'(0) = -1.$$