UNIVERSITY *of* LIMERICK
OLLSCOIL LUIMNIGH

FACULTY OF SCIENCE AND ENGINEERING

DEPARTMENT OF MATHEMATICS & STATISTICS

END OF SEMESTER ASSESSMENT PAPER

MODULE CODE: MA 4005

SEMESTER: Autumn 2008

MODULE TITLE: Engineering Maths T1

DURATION OF EXAMINATION: 2hrs 30mins

LECTURER: Dr. S. Soussi

PERCENTAGE OF TOTAL MARKS: 80%

INSTRUCTIONS TO CANDIDATES:
Answer all questions. All questions carry equal marks. Full marks for correct answers to any 5 questions.

1. (a) Find all partial derivatives of order 2 of the following functions: **8%**
   A. $f(x,y) = ye^{3xy}\cos(xy)$.
   B. $g(x,y) = \dfrac{2x-3y}{x^2-2y}$.

   (b) In an ideal gas, the pressure P, the volume V, the temperature T, and the amount of gas n (in moles) satisfy the following formula: **12%**

$$PV = nRT,$$

   where R is a constant called the gas constant.

   We consider a fixed quantity of gas n_0 enclosed in a box of volume V_0 maintained at a temperature T_0. Starting from that initial state, we deform slightly the box so that its volume is reduces by δV which is supposed to be small (the new volume is $V_0 - \delta V$), and at the same time, we heat the box so that the temperature of the gas is raised by δT (the new temperature is $T_0 - \delta T$).

   i. Write the total differential of P in terms of n, T, V.
   ii. Supposing that all parameters have changed very slightly, find an approximation of the pressure P of the gas in the new state in terms of R, n_0, P_0, V_0, T_0, δV and δT.

2. (a) Find the volume generated when the area under the curve $e^{x/2}$ from $x = 0$ to $x = 1$ is rotated about the x axis. **6%**

   (b) Find the centroid of the previously defined area. **6%**

   (c) Find the moment of inertia of the same area about the x axis. **8%**

3. (a) Evaluate the definite integrals **8%**
   i. $\int_2^3 \frac{dx}{x^2-4x+6}$
   ii. $\int_0^\pi e^{2x}\sin(x)dx$

   (b) Find the general solution of the differential equations **12%**
   i. $y' - 2y = \sin(3x)$
   ii. $y'' - 3y' + 2y = xe^{2x}$

4. (a) Calculate the Laplace transform of $f(t) = te^{-2t}$. **2%**

   (b) Use log tables to find the Laplace transform of the functions **6%**
   i. $f(t) = 3\cosh(t) + 2\sinh(t)$
   ii. $f(t) = U_\pi(t)\cos(t-\pi)$

   (c) Find the inverse Laplace transform of the function $f(s) = \frac{2s+5}{s^2+s-2}$. **6%**

   (d) Use the Laplace transform to find the solution of the boundary value problem **6%**

$$y'' - 2y' + y = 2, \qquad y(0) = 1, \quad y'(0) = -1.$$

5. (a) Which of the following functions are periodic and if so, what is the period? [8%]

   i. $f(x) = x^2$
   ii. $f(x) = e^{\cos(2x+3)}$
   iii. $f(x) = x - [x]$ ($[x]$ is the greatest integer less than or equal to x)
   iv. $\sin(e^x)$.

   (b) Find the Fourier series of period 2π of the function [10%]

$$\begin{cases} f(x) = |x|, & -\pi \leq x < \pi \\ f(x + 2\pi) = f(x) & \forall x \in \mathbb{R} \end{cases}$$

   (c) Taking $x = 0$ deduce an expression for π written in terms of a series. [2%]

6. (a) Find x such that the following matrix is not invertible: [4%]

$$\begin{pmatrix} -1 & 2 & 3 \\ -2 & x & 4 \\ -3 & 2 & 1 \end{pmatrix}$$

   (b) i. Prove that the following matrix is invertible and find its inverse: [10%]

$$\begin{pmatrix} 1 & 1 & 2 \\ 1 & -1 & 3 \\ -2 & 3 & 0 \end{pmatrix}$$

   ii. Use the previous result to solve the following system:

$$\begin{pmatrix} x + y + 2z & = & 2 \\ x - y + 3z & = & -1 \\ -2x + 3y & = & 4 \end{pmatrix}$$

   (c) Find the eigenvalues of the following matrix: [6%]

$$\begin{pmatrix} 2 & 4 & 1 \\ 2 & 2 & 2 \\ 3 & 1 & 3 \end{pmatrix}$$