UNIVERSITY *of* LIMERICK

OLLSCOIL LUIMNIGH

COLLEGE OF INFORMATICS AND ELECTRONICS

DEPARTMENT OF MATHEMATICS & STATISTICS

END OF SEMESTER ASSESSMENT PAPER

MODULE CODE: MA4005 **SEMESTER: Autumn 2007-08**

MODULE TITLE: **Engineering Maths T1** **DURATION OF EXAM: 2.5 hours**

LECTURER: J Leahy **GRADING SCHEME**: **Examination**: 100%

EXTERNAL EXAMINER: Prof J King

INSTRUCTIONS TO CANDIDATES

Answer **One** (1) question from **each** Section A and B and any **three** (3) other questions. Five questions in total. All questions carry equal marks.

SECTION A

Marks

1. **(a)** If $z = e^{xy} + x^2 y$ prove

(i) $$\frac{\partial^2 z}{\partial x \partial y} = \frac{\partial^2 z}{\partial y \partial x}$$

5

(ii) $$\frac{\partial^2 z}{\partial x^2} + \frac{\partial^2 z}{\partial y^2} = (x^2 + y^2)e^{xy} + 2y \; .$$

5

(b) The volume, *V*, of a box is calculated from the formula
10
$V = \ell b h$ using the measurements $\ell = 20\,cm,$
$b = 15\,cm$ and $h = 10\,cm.$ The maximum error in each measurement
is $0.1\,cm.$ Find, using partial derivatives the maximum error in the
calculated value of *V*.

2. **(a)** Evaluate the integrals

(i) $$\int_{7/4}^{2} (4x - 7)^{2007}\, dx$$

3

(ii) $$\int_{-1}^{1} \frac{dx}{x^2 + 4x + 40} \; .$$

3

(b) Find the area of the region bounded by the curves **6**

$y = x^3, \ y = 0, \ x = 1.$

(c) Find the centroid of the area in (b).

8

Marks

3. (a) Find the general solution of each of the differential equations

(i) $\dfrac{dy}{dx} + 5y = e^{2x}$ **6**

(ii) $y'' - 3y' + 2y = \sin x + \cos x$. **6**

(b) A body moves so that its velocity at any time *t* is directly proportional to its distance *x* from a fixed point. Its equation of motion is

$\dfrac{dx}{dt} = kx,$ k constant.

Find an expression for *x* in terms of *t* given that $x = e$ at $t = 0$.

8

4. (a) Calculate from the definition the Laplace transform of the function **4**

$f(t) = 2t$.

(b) Use the tables to find the Laplace transform of the functions **2**

(i) $f(t) = t^4 + \sinh 5t$

(ii) $f(t) = e^{4t}\sin 2t$.

2

(c) Find the inverse Laplace transform of the function

$$F(s) = \frac{2 + 3s - s^2}{(s+1)(s^2+1)}.$$

6

(d) Use the Laplace transform to find the solution of the

6

boundary value problem

$$y' + y = t, \quad y(0) = 1.$$

Marks

5. (a) Find the period of the following periodic functions

(i) $f(x) = \sin(2x + 1)$ **2**

(ii) $f(x) = \cos^2 x$. **2**

Determine if either of these functions is even or odd.

2

(b) The function $f(x)$ is periodic of period 2π and is defined over the interval $[-\pi, \pi]$ by

$$f(x) = \begin{cases} -3 & -\pi < x \leq 0 \\ 3 & 0 < x \leq \pi \end{cases}, \quad f(x+2\pi) = f(x).$$

Find its Fourier series.

14

SECTION B

6. **(a)** Find one non-trivial solution of the homogeneous system of **6**

linear equations

$$x + y - 2z = 0$$
$$2x - y + z = 0$$
$$-x + 5y - 8z = 0.$$

(b) Find the inverse of the matrix
7

$$\begin{pmatrix} 1 & 1 & 1 \\ 2 & -1 & -1 \\ 3 & 1 & -1 \end{pmatrix}$$

and hence solve the system
7

$$\begin{aligned} x + y + z &= 1 \\ 2x - y - z &= 2 \\ 3x + y - z &= 5. \end{aligned}$$

Q.7 **(a)** State the axioms for a vector space.
4

(b) Show the set *R* of all real numbers with the usual operations of addition and multiplication by a scalar is a vector space. 6

(c) Determine which of the following sets of vectors are linearly independent in R^3

(i) $\underline{v}_1 = (1,0,1)$, $\underline{v}_2 = (0,1,1)$, $\underline{v}_3 = (1,1,0)$

(ii) $\underline{v}_1 = (1,1,1)$, $\underline{v}_2 = (1,-2,3)$, $\underline{v}_3 = (3,0,5)$
. 5

(d) Show that the vectors
$\underline{v}_1 = (1,2)$ and $\underline{v}_2 = (-1,4)$
span R^2.

Q.8 Find the eigenvalues and corresponding eigenvectors of the matrix

$$A = \begin{pmatrix} 2 & 0 & 0 \\ 0 & 1 & -1 \\ 0 & 0 & 1 \end{pmatrix}.$$ 14

Determine the Rank and nullity of A.

6