UNIVERSITY *of* LIMERICK

OLLSCOIL LUIMNIGH

COLLEGE OF INFORMATICS AND ELECTRONICS

DEPARTMENT OF MATHEMATICS & STATISTICS

END OF SEMESTER ASSESSMENT PAPER

MODULE CODE: MA4005 **SEMESTER: Autumn 2006-07**

MODULE TITLE: **Engineering Maths T1** **DURATION OF EXAM: 2.5 hours**

LECTURER: J Leahy **GRADING SCHEME**: **Examination**: 100%

EXTERNAL EXAMINER: Prof J King

INSTRUCTIONS TO CANDIDATES

Answer **One** (1) question from **each** Section A and B and any **three** (3) other questions. Five questions in total. All questions carry equal marks.

SECTION A

Marks

1. a) Find all second order partial derivatives of the following functions

(i) $w = x^2 + xy^2 + xyz^2$ **(ii)** $z = e^{x^2 y}$

8

b)

The area A of a rhombus is calculated using the formula

$A = b^2 \sin C$

Using the measured values of 4m and 45º for b and C respectively. Find using partial differentiation the maximum error in the area as calculated if there is a maximum error of 0.3cm in the measurement of b and 0.5º in the measurement of C.

12

2. a) Evaluate the integrals

(i) $\int_0^1 \frac{dx}{(x-2)^3}$ (ii) $\int_2^{4\sqrt{3}-2} \frac{dx}{x^2+4x+20}$

8

b)

Find the volume generated when the area between the curve

$y = 1 + x^3$ and the x – axis from x = 0 to x = 1 is rotated about the x-axis.

6

c) Find the moment of inertia about the x – axis of the area in (b).

6

3. a) Find the general solution of each of the differential equations

(i) $\frac{dy}{dx}$ - y sin x = $e^{-\cos x}$

(ii) $y'' + 2y = t^2 - 1$,

12

b)

The vertical motion of a buoy of mass m and cross – sectional area A floating in water of density ρ is given by the equation

$\frac{d^2z}{dt^2} + \frac{\rho A g}{m} z = g$ where g> 0 is the gravitational constant.

Express z as a function of t.

8

Marks

4. a) Calculate, from the definition, the Laplace transform of the function

$$f(t) = 1 - e^{4t}$$

4

b)

Use the tables to find the Laplace transform of the functions

(i) $f(t) = 3 \cosh 4t - 2 \sin 2t$
(ii) $f(t) = t^4 e^{-2t}$

4

c) Find the inverse Laplace transform of the function

$$F(s) = \frac{s+2}{(4s+1)(s-1}.$$

6

d)

Use the Laplace transform to find the solution of the boundary value problem

$$\frac{dy}{dt} - 2y = 5e^{t} \qquad y(o) = 0$$

6

5. Find the Fourier series of period 2π of the function

$$f(x) = \begin{cases} x & 0 \le x \le \pi \\ x & -\pi < x < 0 \end{cases} \qquad f(x + 2\pi) = f(x).$$

16

Use your answer to find an expression for π^2

4

SECTION B

Marks

6. a) Prove (a + b + c) is a factor of the determinant

$$\begin{vmatrix} a & b & c \\ b & c & a \\ c & a & b \end{vmatrix}$$

3

b)

Show the system of linear equations

$x - y + z = 2$
$2x + y - z = 1$
$4x - y + z = 5$
has an infinite number of solutions and find two solutions.

6

c)

Find the inverse of the matrix

$$A: \begin{pmatrix} 2 & 1 & 1 \\ -1 & 1 & 3 \\ 4 & 1 & 2 \end{pmatrix}$$

6

and hence solve the system

$$\begin{aligned} 2x + y + z &= 3 \\ -x + y + 3z &= -7 \\ 4x + y + 2z &= 5 \end{aligned}$$

5

7. a) State the axioms for a vector space.

4

b)

Show the set of all complex numbers a + bi, a, b ε R, i = $\sqrt{-1}$, with the usual addition and multiplication by a scalar is a vector space.

6

c) Show the vectors $\underline{u}_1$ = (1, 1, 0), $\underline{u}_2$ = (2, 0, -3) and $\underline{u}_3$ = (0, 1, 5) are linearly independent in R^3.

5

d)

Determine if the set of vectors

$\underline{v}_1$ = (-1, -1, 0)
$\underline{v}_2$ = (-1, 0, -1)

$\underline{v}_3 = (0, -1, -1)$

span R^3

5

Marks

8. If $A = \begin{pmatrix} 2 & 9 & 0 \\ 1 & 2 & 0 \\ 0 & 0 & 3 \end{pmatrix}$ is a matrix find

(i) the eigenvalues of A

7

(ii) the eigenvectors of A corresponding to any one of its eigenvalues.

6

(iii) the rank and mullity of A.

7