UNIVERSITY *of* LIMERICK
OLLSCOIL LUIMNIGH

COLLEGE OF INFORMATICS AND ELECTRONICS

DEPARTMENT OF MATHEMATICS & STATISTICS

END OF SEMESTER ASSESSMENT PAPER

MODULE CODE: **MA4005**   **SEMESTER: Autumn 2005-06**

MODULE TITLE: **Engineering Maths T1**   **DURATION OF EXAM: 2.5 hours**

LECTURER: **J. Leahy**   **GRADING SCHEME**:
                                                Examination: **100%**

EXTERNAL EXAMINER: Prof J King

INSTRUCTIONS TO CANDIDATES

Answer **One** (1) question from **each** Section A and B and any **three** (3) other questions –
Five questions in total. All questions carry equal marks.

SECTION A

Marks

1. a) Find all second order partial derivatives of the following functions

(i) $z = \dfrac{x^2}{y+1}$ (ii) $z = \cos(2xy)$

8

b) The pressure, P, of an ideal gas is calculated from the formula
$$P = \frac{kT}{V}$$
where T is the temperature, V the volume and k a constant, using the measured values T = -20°K and V = 1000 cm³. If the maximum error in T is 0.05°K and in V is 2cm³ find using partial differentiation and taking k = 1, the maximum error in P as calculated.

12

2. a) Evaluate the definite integrals

(i) $\displaystyle\int_{2}^{3}(x-3)^{2006}\,dx$

8

(ii) $\displaystyle\int_{3}^{4}\frac{dx}{x^2+x-2}$

b)

Find the volume generated when the area under the curve $y = 2x^3$ from x = 0 to x = 1 is rotated about the x axis.

6

c) Find the centroid of the area in (b).

6

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Marks

3. a) Find the general solution of the differential equations

(i) $\dfrac{dy}{dx} - 3y = e^{2x}$

(ii) $y'' - 3y' + 2y = 5 \cos 2x$

12

b) The velocity v of a pendulum oscillating under the force of gravity satisfies the differential equation

$v \dfrac{dv}{dx} = -k^2 x$

where x is the displacement and k is a constant. Find v in terms of x if $v = 10$m/sec when $x = 0$.

8

4. a) Calculate from the definition the Laplace transform of the function

$f(t) = 1 + e^t$

4

b) Use the tables to find the Laplace transform of the functions

(i) $f(t) = 2 \cosh t + 3 \sinh t$

(ii) $f(t) = u_\pi(t) \sin(t - \pi)$

4

c) Find the inverse Laplace transform of the function

$F(s) = \dfrac{7s + 11}{s^2 + s - 6}$

6

d) Use the Laplace transform to find the solution of the boundary value problem

$$y'' - y' = 1, \quad y(o) = y'(o) = 1$$

6

5. Find the Fourier series of period 2π of the function

$$f(x) = -x, \qquad -\pi < x \le \pi, \qquad f(x + 2\pi) = f(x).$$

12

Sketch the graph of f(x) and use the series to find an expression for π **3 + 5**

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SECTION B

Marks

6. a) If the determinant

$$\begin{vmatrix} 9 & \cdot 3 & 24 \\ x & 1 & \cdot 8 \\ 5 & 0 & 7 \end{vmatrix} : 0$$

3

find x

b) Show that the matrix

$$\begin{vmatrix} 1 & 2 & \cdot 4 \\ \cdot 1 & 3 & 6 \\ 1 & 12 & 0 \end{vmatrix}$$

3

has no inverse.

c) The currents i_1, i_2, i_3 of a circuit satisfy the system
$$2i_1 + i_2 - i_3 = 8$$
$$i_1 - i_2 + i_3 = -5$$
$$3i_1 + 2i_2 = 9$$
Use matrices to evaluate i_1, i_2, i_3.

14

7. a) Define a vector space.

4

b) Show the set of all linear forms $ax + b$ forms a vector space.

6

c) Determine if the set of vectors
$v_1 = (-1, 0, 2)$, $v_2 = (3, 1, 4)$, $v_3 = (1, 1, 8)$ are linearly independent.

5

d) Determine if the set of vectors
$u_1 = (1, 0, 0)$, $u_2 = (1, 2, 0)$, $u_3 = (1, 2, 3)$ span R^3.

5

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Marks

8. Show that 2 is an eigenvalue of the matrix

$$A = \begin{vmatrix} 1 & 2 & 2 \\ 0 & 2 & 1 \\ \cdot 1 & 2 & 2 \end{vmatrix}$$

and find the other eigenvalue.

7

Find the eigenvectors corresponding to the eigenvalue 2.

7

Find the rank and nullity of A.

6