UNIVERSITY *of* LIMERICK
OLLSCOIL LUIMNIGH

COLLEGE OF INFORMATICS AND ELECTRONICS

DEPARTMENT OF MATHEMATICS & STATISTICS

END OF SEMESTER ASSESSMENT PAPER

MODULE CODE: MA4005 **SEMESTER: Autumn 2004-05**

MODULE TITLE: **Engineering Maths T1** **DURATION OF EXAM: 2.5 hours**

LECTURER: J. Leahy **GRADING SCHEME**: **Examination**: **100%**

EXTERNAL EXAMINER: Prof J. King

INSTRUCTIONS TO CANDIDATES

Answer **One** (1) question from **each** Section A and B and any **three** (3) other questions – Five questions in total. All questions carry equal marks.

MA4005 Engineering Maths T1

Marks

SECTION A

1. a) Find all second partial derivatives of the following functions

(i) $z = \frac{x^2}{y^5}$ **(ii)** $z = e^{2x} + \sin(xy)$.

8

b) The volume of a cylinder is calculated from the formula

$V = \pi r^2 h$

using the measurements r =20cm, h = 30cm. The maximum error in the measurements of r and h is 0.05cm. Find using partial derivatives the maximum error in the calculated value of V.

12

2. a) Evaluate the integrals

(i) $\int \frac{dx}{\sqrt{2 - 3x^2}}$ **(ii)** $\int \frac{dx}{x^2 - 3x + 2}$

10

b) Find the centroid of the area bounded by the curve $y = x^2 + 1$, the x-axis, and the verticals x = 0 and x = 2.

10

3 a) Find the general solutions of the differential equations

(i) $\frac{dy}{dx} + xy = xy^2$ **(ii)**

$$y'' + y = t^3 + 1$$

12

b) The current in a circuit is given by

$$L\frac{di}{dt}+Ri=E$$

where *L*, *R* and *E* are constants. Find the general solution for *i* in terms of t and the particular solution for which $i = i_o$ at $t = 0$. Show the terminal value of the current is $\frac{E}{R}$.

8

Marks

4. a) Calculate from the definition the Laplace transform of the function

$$f(t) = 2e^{-3t}$$

4

b) Use the tables to find the Laplace transform of the functions

(i) $f(t) = 5\cos 2t + \sin 3t$

(ii) $f(t) = t^3 e^{2t}$

4

c) Find the inverse Laplace transform of the function

$$F(s) = \frac{2s-1}{4s^2+3s-1}$$

6

d) Use the Laplace transform to find the solution of the boundary value problem.

$$2\frac{dy}{dt} - y = 2e^{2t}, \qquad y(0) = 3.$$

6

5. Find the Fourer series of period 2π of the function

$$f(x) = \begin{cases} 1 + x & 0 \le x \le \pi \\ 1 - x & -\pi \le x \le 0 \end{cases}$$

$f(x + 2\pi) = f(x)$

17

Use your answer to find an expression for π^2.

3

Marks

SECTION B

6. a) Evaluate the determinant

$$\begin{vmatrix} 3 & 5 & 8 \\ 13 & 21 & 34 \\ 55 & 89 & 144 \end{vmatrix}$$

3

b) Show the system of linear equations

x + y + z = 1
2x – 3y + 2z = -3
4x – y + 4z = -1
has an infinite number of solutions and find two solutions.

7

c) Find the inverse of the matrix

$$A = \begin{pmatrix} 2 & 3 & 1 \\ 4 & -1 & 2 \\ 1 & 1 & 1 \end{pmatrix}$$

5

and hence solve the system

2x + 3y+ z = 9
4x – y + 2z = 4
x + y + z = 4.

5

Marks

7. a) Define a vector space.

4

b) Show the set of all 2 x 2 matrices with real entries together with the operations of matrix addition and scalar multiplication is a vector space.

4

c) Show the vectors $u_1 = (1,0,3)$, $u_2 = (0,-1,4)$ and $u_3 = (2,3,-6)$ are linearly dependent in R^3.

6

d) Determine if the set of vectors

$\underline{v}_1 = (1, 1, 2)$

$\underline{v}_2 = (1, 0, 1)$

$\underline{v}_3 = (2, 1, 3)$

span R^3.

6

8. If A = $\begin{pmatrix} 2 & 1 & 0 \\ 3 & 2 & 0 \\ 0 & 0 & 4 \end{pmatrix}$ is a matrix

find

(i) the eigenvalues of A

7

(ii) the eigenvectors of A corresponding to any one of its eigenvalues.

7

and **(iii)** the rank and nullity of A.

6