UNIVERSITY *of* LIMERICK
OLLSCOIL LUIMNIGH

COLLEGE OF INFORMATICS AND ELECTRONICS

DEPARTMENT OF MATHEMATICS & STATISTICS

END OF SEMESTER ASSESSMENT PAPER

MODULE CODE: MA4005 **SEMESTER: Autumn 2003/04**

MODULE TITLE: Engineering Maths T1 **DURATION OF EXAM: 2.5 hours**

LECTURER: J. Leahy **GRADING SCHEME: Examination: 100%**

EXTERNAL EXAMINER: Prof. J.D. Gibbon

INSTRUCTIONS TO CANDIDATES

Answer **<u>One</u>** (1) question from each Section A and B and any three other questions – Five questions in total. All questions carry equal marks

Engineering Maths T1

Marks

SECTION A

Q.1 **(a)** Find all second partial derivatives of the following functions

(i) $z = x^2y^3$ **(ii)** $z = x \sin(xy)$ **8**

(b) The surface area of a closed cone is calculated from the formula

$S = \pi r(r + l)$

using the measured values of 2m and 3m for r and l respectively. Find using partial differentiation the maximum error in the area as calculated if there is a maximum error of 0.2cm in the measurement of each of r and l. **12**

Q.2 **(a)** Evaluate the integrals

(i) $\int \frac{dx}{4x^2+1}$ **(ii)** $\int \frac{dx}{x^2+6x-2}$

(iii) $\int \frac{x+\cos 2x}{x^2+\sin 2x}\,dx$ **12**

(b) Find the perimeter of the area bounded by the curve y = coshx, the x-axis, the y-axis and the line x = ln3. **8**

Q.3 **(a)** Find the general solutions of the differential equations

(i) $\frac{dy}{dx}+2xy = e^{1-x^2}$ **(ii)** $y'' - y' = \sin x - \cos x$

12

(b) The vertical motion of a buoy of mass m and cross-sectional area A floating in water of density ρ is given by the equation

$$\frac{d^2z}{dt^2}+\frac{\rho A g}{m}z = g$$

Express z as a function of t. **8**

Marks

Q.4 **(a)** Calculate from the definition the Laplace transform of the function

$f(t) = 1 - e^{-2t}$ **4**

(b) Use the tables to find the Laplace transform of the functions

(i) $f(t) = 2 \cosh 2t - 3 \cos 3t$

(ii) $f(t) = e^{3t} \sin 5t$ **4**

(c) Find the inverse Laplace transform of the function

$$F(s) = \frac{4 + s^2 - 2s^3}{s^2(s^2 + 4)}$$ **6**

(d) Use the Laplace transform to find the solution of the boundary value problem

$$y'' - y = e^{-t} \qquad y(0) = \frac{1}{2} \qquad y'(0) = 0$$ **6**

Q.5 A periodic function f(x) with period 2π is defined by

$$f(x) = \begin{cases} 2 & 0 \le x \le \pi \\ -2 & -\pi \le x < 0 \end{cases}$$

Sketch the graph of f(x) for $-2\pi \le x \le 2\pi$ **3**
and obtain a Fourier Series expansion of the function. **14**

Use the series to find an expression for π. **3**

Marks

SECTION B

Q.6 **(a)** Prove abc is a factor of the determinant

$$\begin{vmatrix} a & b & c \\ a^2 & b^2 & c^2 \\ a^3 & b^3 & c^3 \end{vmatrix}$$ **3**

(b) Show the system of linear equations

x + y = 1
y – z = 2
z + x = -1

has an infinite number of solutions and find two solutions. **7**

(c) Find the inverse of the matrix

$$A = \begin{pmatrix} 3 & 1 & 1 \\ 1 & 1 & -1 \\ 5 & -1 & -1 \end{pmatrix}$$ **5**

and hence solve the system

3x + y + z = 0
x + y - z = 2
5x - y - z = 0. **5**

Q.7 **(a)** Define a vector space **4**

(b) Show the set of all 2 x 2 diagonal matrices with real entries together with the operations of matrix addition and scalar multiplication is a vector space. **4**

(c) Show the vectors $\underline{u}_1 = (1,2,3)$, $\underline{u}_2 = (2,-1,1)$ and $\underline{u}_3 = (0\ 1\ 2)$ are linearly independent in R^3. **6**

(d) Enlarge the set of vectors $\{(1,0,1),(0,2,0)\}$ to form a basis for R^3.
6

Marks

Q.8 $A = \begin{pmatrix} 1 & 2 & 2 \\ 0 & 2 & 1 \\ -1 & 2 & 2 \end{pmatrix}$ is a matrix with an eigenvalue $\lambda_1 = 2$.

Find

(i) the other eigenvalues of A. **6**

(ii) the eigenvectors of A corresponding to any one of its eigenvalues. **6**

(iii) the rank and nullity of A. **8**

TABLE OF LAPLACE TRANSFORMS

f(t)	F(s)
1	$\frac{1}{s}$
t^n	$\frac{n!}{s^{n+1}}$
t^α	$\frac{\Gamma(a+1)}{s^{\alpha+1}}$
e^{at}	$\frac{1}{s-a}$
$\sin at$	$\frac{a}{s^2+a^2}$
$\cos at$	$\frac{s}{s^2+a^2}$
$\sinh at$	$\frac{a}{s^2-a^2}$
$\cosh at$	$\frac{s}{s^2-a^2}$
$e^{at} f(t)$	$F(s-a)$

$u_a(t)$	$\frac{e^{-as}}{s}$
$u_a(t)f(t-a)$	$e^{-as}F(s)$
$tf(t)$	$-F'(s)$
$t^n f(t)$	$(-1)^n F^{(n)}(s)$
$f'(t)$	$sF(s)-f(0)$
$f''(t)$	$s^2F(s)-sf(0)-f'(0)$
$f^{(n)}(t)$	$s^nF(s)-s^{n-1}f(0)-\ldots-f^{(n-1)}(0)$
$\int_0^t f(u)\,du$	$\frac{1}{s}F(s)$
$\frac{f(t)}{t}$	$\int_s^\infty F(u)\,du$